Math 444 Name:___________________

Exam 1 S. S. #:___________________

Each problem is worth 20 points.

Do any five of the six problems. Circle the five you problems you turn in!

1. Assume the following constraints are satisfied:

$$\begin{cases} 2x_1 - x_2 \geq 2 \\ x_1 + x_2 \leq 6 \\ x_1 + 2x_2 = 8 \\ x_1, x_2 \geq 0 \end{cases}.$$

(a) Use the graphical method to determine the maximizer and the minimizer of $f(x_1,x_2) = 3x_1 - 4x_2$.

(b) Using the same constraints, convert the constraints to the form of a primal problem.

2. Use the simplex algorithm to solve the problem:

$$\max x_1 - 2x_2 + 3x_3 \text{ when } \begin{cases} x_1 + x_3 \leq 4 \\ x_2 + x_3 \leq 5 \\ x_1, x_2, x_3 \geq 0 \end{cases}.$$

Make sure that each step employed is the step suggested by the simplex algorithm.

3. Write down but do not solve the dual problem of the following primal problem:

$$\max x_1 - x_3 + 2x_4 \text{ when } \begin{cases} 2x_2 - x_4 \leq 12 \\ 3x_1 - x_2 + 4x_5 \leq 20 \\ x_2 - x_3 + x_4 + x_5 \leq 25 \\ x_i \geq 0, i \in \{1, \ldots, 5\} \end{cases}$$

4. Complete the first phase of the two-phase method to find a feasible point of the primal problem with constraints:

$$\begin{cases} x_1 - 2x_2 + 3x_3 \leq -6 \\ 2x_1 + x_2 \leq 12 \\ x_1, x_2, x_3 \geq 0 \end{cases}.$$

Make sure that each step employed is the step suggested by the first phase of the two-phase method.

5. Write down the primal problem corresponding to the following dual problem. Apply the simplex method to the primal problem. From the final tableau, find the minimizer $\hat{\lambda}^T = \begin{bmatrix} \hat{\lambda}_1 & \hat{\lambda}_2 & \hat{\lambda}_3 \end{bmatrix}$:

$$\min\ 4\lambda_1 + 3\lambda_2 + 2\lambda_3 \text{ when } \begin{cases} 2\lambda_1 + \lambda_2 \geq 2 \\ \lambda_3 \geq 1 \\ \lambda_1 + \lambda_3 \geq 6 \\ \lambda_1, \lambda_2, \lambda_3 \geq 0 \end{cases}$$

6. The problem:

$$\max\ 25x_1 + 30x_2 + 18x_3 \text{ when } \begin{cases} 2x_1 + 3x_2 + 4x_3 \leq 60 \\ 3x_1 + x_2 + 5x_3 \leq 45 \\ x_1 + 2x_2 + x_3 \leq 50 \\ x_1, x_2, x_3 \geq 0 \end{cases}$$

has a maximizer $\hat{x}_1 = \frac{75}{7}, \hat{x}_2 = \frac{90}{7}, \hat{x}_3 = 0$. Use nothing but complementary slackness to find the dual solution $\hat{\lambda}$.