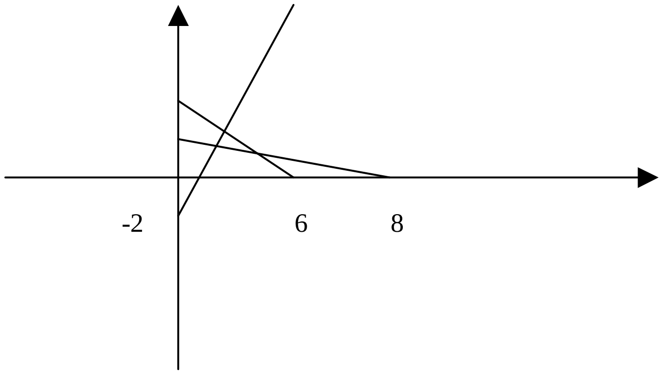
1. (a)



The extreme points of the feasible set are given by $\begin{cases} x_1 + 2x_2 = 8 \\ x_1 + x_2 = 6 \end{cases}$ and $\begin{cases} x_1 + 2x_2 = 8 \\ 2x_1 - x_2 = 2 \end{cases}$. The points are $(4,2)$ and $\left(\frac{12}{5}, \frac{14}{5}\right)$. The values are 4 and -4 respectively.

(b) $\begin{cases} -2x_1 + x_2 \le -2 \\ x_1 + x_2 \le 6 \\ x_1 + 2x_2 \le 8 \\ -x_1 - 2x_2 \le -8 \\ x_1, x_2 \ge 0 \end{cases}$

2. Initial tableau:

$$\begin{array}{ccccc|c} 1 & 0 & 1 & 1 & 0 & 4 \\ 0 & 1 & 1 & 0 & 1 & 5 \\ \hline 1 & -2 & 3 & 0 & 0 & 0 \end{array}.$$

Final tableau:

$$\begin{array}{ccccc|c} 1 & 0 & 1 & 1 & 0 & 4 \\ -1 & 1 & 0 & -1 & 1 & 1 \\ \hline -2 & -2 & 0 & -3 & 0 & -12 \end{array}.$$

The solution is given by $\hat{x}_1 = 0, \hat{x}_2 = 0, \hat{x}_3 = 4$, the maximum value is 12.

3. $n = 5, m = 3$, $c^T = \begin{bmatrix} 1 & 0 & -1 & 2 & 0 \end{bmatrix}$, $b^T = \begin{bmatrix} 12 & 20 & 25 \end{bmatrix}$, $A = \begin{bmatrix} 0 & 2 & 0 & -1 & 0 \\ 3 & -1 & 0 & 0 & 4 \\ 0 & 1 & -1 & 1 & 1 \end{bmatrix}$.

The dual $\min \lambda^T b$ when $\begin{cases} A^T \lambda \ge c \\ \lambda \ge 0 \end{cases}$ is here given by

$$\min 12\lambda_1 + 20\lambda_2 + 25\lambda_3 \text{ when } \begin{cases} 3\lambda_2 \ge 1 \\ 2\lambda_1 - \lambda_2 + \lambda_3 \ge 0 \\ -\lambda_3 \ge -1 \\ -\lambda_1 + \lambda_3 \ge 2 \\ 4\lambda_2 + \lambda_3 \ge 0 \\ \lambda_1, \lambda_2, \lambda_3 \ge 0 \end{cases}.$$

4. Using two bottom rows the initial tableau is given by:
$$\begin{array}{cccccc|c} -1 & 2 & -3 & -1 & 0 & 1 & 6 \\ 2 & 1 & 0 & 0 & 1 & 0 & 12 \\ \hline 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ \hline -1 & 2 & -3 & -1 & 0 & 0 & 6 \end{array}.$$

The final tableau of first phase is given by:
$$\begin{array}{cccccc|c} -\frac{1}{2} & 1 & -\frac{3}{2} & -\frac{1}{2} & 0 & \frac{1}{2} & 3 \\ \frac{5}{2} & 0 & \frac{3}{2} & 1 & 1 & -1 & 9 \\ \hline 0 & 0 & 0 & 0 & 0 & -\frac{1}{2} & 0 \end{array}.$$

The corresponding feasible point is given $x_1 = 0, x_2 = 3, x_3 = 0, y_1 = 0, y_2 = 9$.

5. The initial tableau of the primal problem is given by:
$$\begin{array}{cccccc|c} 2 & 0 & 1 & 1 & 0 & 0 & 4 \\ 1 & 0 & 0 & 0 & 1 & 0 & 3 \\ 0 & 1 & 1 & 0 & 0 & 1 & 2 \\ \hline 2 & 1 & 6 & 0 & 0 & 0 & 0 \end{array}$$

First pivot:
$$\begin{array}{cccccc|c} 2 & -1 & 0 & 1 & 0 & -1 & 2 \\ 1 & 0 & 0 & 0 & 1 & 0 & 3 \\ 0 & 1 & 1 & 0 & 0 & 1 & 2 \\ \hline 2 & -5 & 0 & 0 & 0 & -6 & -12 \end{array}.$$

Final pivot
$$\begin{array}{cccccc|c} 1 & -\frac{1}{2} & 0 & \frac{1}{2} & 0 & -\frac{1}{2} & 1 \\ 0 & \frac{1}{2} & 0 & -\frac{1}{2} & 1 & \frac{1}{2} & 2 \\ 0 & 1 & 1 & 0 & 0 & 1 & 2 \\ \hline 0 & -4 & 0 & -1 & 0 & -5 & -14 \end{array}$$

The dual global minimum is given by $\hat{\lambda}_1 = 1, \hat{\lambda}_2 = 0, \hat{\lambda}_3 = 5$. The dual slack is given by $\hat{\mu}_1 = 0, \hat{\mu}_2 = 4, \hat{\mu}_3 = 0$.

6. First determine the slack: $\hat{y}_1 = 0, \hat{y}_2 = 0, \hat{y}_3 > 0$. Use complementary slackness and conclude that $\hat{\lambda}_3 = 0$. From $\hat{x}_1 > 0, \hat{x}_2 > 0$ conclude that $\hat{\mu}_1 = 0, \hat{\mu}_2 = 0$.

It follows that $\begin{cases} 2\hat{\lambda}_1 + 3\hat{\lambda}_2 = 25 \\ 3\hat{\lambda}_1 + \hat{\lambda}_2 = 30 \end{cases}$.

The solution of the system is given by $\begin{bmatrix} \hat{\lambda}_1 \\ \hat{\lambda}_2 \end{bmatrix} = \dfrac{1}{2 \cdot 1 - 3 \cdot 3} \begin{bmatrix} 1 & -3 \\ -3 & 2 \end{bmatrix} \begin{bmatrix} 25 \\ 30 \end{bmatrix} = \dfrac{1}{7} \begin{bmatrix} 65 \\ 15 \end{bmatrix}$.