

Math 444

Name: _____

Exam 2

S. S. #: _____

Each problem is worth 20 points.

Do any five of the six problems. Circle the five you problems you turn in!

1. Suppose $A = \begin{bmatrix} 1 & 2 \\ 1 & 1 \\ 3 & 2 \end{bmatrix}$.

What is the value of v if the final tableau is

$$\begin{array}{cccc|c} 0 & 1 & 1 & -1 & 0 & 5 \\ 1 & 0 & -1 & 2 & 0 & 4 \\ 0 & 0 & 1 & -4 & 1 & 2 \\ \hline 0 & 0 & -1 & -1 & 0 & -v \end{array} ?$$

2. Suppose $b = \begin{bmatrix} 6 \\ 9 \\ 12 \end{bmatrix}$ yields the final tableau:

$$\begin{array}{cccccc|c} 0 & 0 & \frac{17}{3} & 1 & -\frac{1}{3} & 2 & 27 \\ 1 & 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 & 3 \\ 0 & 1 & 3 & 0 & 0 & 1 & 12 \\ \hline 0 & 0 & -\frac{88}{3} & 0 & -\frac{10}{3} & -12 & -174 \end{array} .$$

Assume only one of the entries in b is allowed to vary, and the variation is such that the final tableau is still applicable.

- (a) Determine the three intervals of permissible variations.
(b) What is the "new" maximum value attained by the largest permissible variation of 9?

3. Suppose $c = \begin{bmatrix} 10 \\ 20 \end{bmatrix}$ yields the final tableau:

$$\begin{array}{cccc|c} 1 & 1 & 1 & 0 & 10 \\ 3 & 0 & -2 & 1 & 0 \\ \hline -10 & 0 & -20 & 0 & -250 \end{array}$$

- (a) The first variable is non-basic. How much can $c_1 = 10$ be increased and still have the same basic variables?
(b) The second variable is basic. How much can $c_2 = 20$ be decreased and still have the same basic variables?

4. The profit / item of three products is 1, 4, 3 respectively. The relevant linear programming problem in primal form is

$$\max x + 4y + 3z \text{ when } \begin{cases} 2x + 2y + z \leq 4 \\ x + 2y + 2z \leq 6 \\ x, y, z \geq 0 \end{cases}$$

The corresponding final tableau is given by:

$$\begin{array}{ccccc|c} \frac{3}{2} & 1 & 0 & 1 & -\frac{1}{2} & 1 \\ -1 & 0 & 1 & -1 & 1 & 2 \\ \hline -2 & 0 & 0 & -1 & -1 & -10 \end{array}$$

As seen from this tableau, the first product is not to be produced.

Determine how much more profit / item is needed to give production of the first product consideration.

5. Suppose the payoff matrix is given by

$$P_{II} \begin{matrix} P_I \\ \begin{bmatrix} 4 & 0 & 1 & 2 \\ 1 & 5 & 3 & 5 \end{bmatrix} \end{matrix}$$

- (a) Eliminate dominated plans.
- (b) Use graphs to find the preferred probability distribution for player P_I .
- (c) Use the same graphs to find the preferred probability distribution for player P_{II} .

6. Suppose the payoff matrix is given by $\begin{bmatrix} 0 & 2 \\ 2 & -1 \\ 1 & 0 \end{bmatrix}$.

- (a) Setup a primal problem that yields a solution to the game.
- (b) The standard alternative produces the final tableau:

$$\begin{array}{ccccc|c} 0 & 1 & \frac{2}{7} & -\frac{1}{7} & 0 & \frac{1}{7} \\ 1 & 0 & \frac{1}{14} & \frac{2}{7} & 0 & \frac{3}{14} \\ 0 & 0 & -\frac{5}{14} & -\frac{4}{7} & 1 & \frac{1}{14} \\ \hline 0 & 0 & -\frac{3}{14} & -\frac{1}{7} & 0 & -\frac{5}{14} \end{array}$$

Use this tableau to compute the two preferred probability distributions.

- (c) Give a payoff matrix of a fair game that has the same preferred probability distributions.