1. $\hat{\lambda}^T = \begin{bmatrix} 1 & 1 & 0 \end{bmatrix}$, $c^T - \hat{\lambda}^T A = c^T - \begin{bmatrix} 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 1 & 1 \\ 3 & 2 \end{bmatrix} = c^T - \begin{bmatrix} 2 & 3 \end{bmatrix} = \begin{bmatrix} 0 & 0 \end{bmatrix}$

$v = c^T \hat{x} = \begin{bmatrix} 2 & 3 \end{bmatrix} \begin{bmatrix} 4 \\ 5 \end{bmatrix} = 23$

2. (a) $B^{-1}\left(b + \Delta_b\right) = B^{-1}b + B^{-1}\Delta_b = \begin{bmatrix} 27 \\ 3 \\ 12 \end{bmatrix} + B^{-1}\Delta_b \geq 0$ leads to $27 + \Delta_{b_1} \geq 0$,

$27 - \frac{1}{3}\Delta_{b_2} \geq 0$, $3 + \frac{1}{3}\Delta_{b_2} \geq 0$, $27 + 2\Delta_{b_3} \geq 0$, $12 + \Delta_{b_3} \geq 0$ so

$-27 \leq \Delta_{b_1}$, $-9 \leq \Delta_{b_2} \leq 81$, $-12 \leq \Delta_{b_3}$

(b) $\hat{\lambda}^T\left(b + \Delta_b\right) = \hat{\lambda}^T b + \hat{\lambda}^T \Delta_b = 174 + \begin{bmatrix} 0 & \frac{10}{3} & 12 \end{bmatrix} \begin{bmatrix} 0 \\ 81 \\ 0 \end{bmatrix} = 444$

3. (a) The only part of the tableau that changes is the

$(c + \Delta_c)^T - c_B^T B^{-1} A = c^T - c_B^T B^{-1} A + \Delta_c^T = \begin{bmatrix} -10 & 0 \end{bmatrix} + \begin{bmatrix} \Delta & 0 \end{bmatrix} \leq 0$ so the increase allowed is at most 10.

(b) x_2, y_2 are basic variables, so $c_B^T = \begin{bmatrix} 20 & 0 \end{bmatrix}$. This time

$(c + \Delta_c)^T - \left(c_B^T + \Delta_{c_B}^T\right) B^{-1} A = c^T - c_B^T B^{-1} A + \Delta_c^T - \Delta_{c_B}^T B^{-1} A =$

$\begin{bmatrix} -10 & 0 \end{bmatrix} + \begin{bmatrix} 0 & \Delta \end{bmatrix} - \begin{bmatrix} \Delta & 0 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 3 & 0 \end{bmatrix} = \begin{bmatrix} -10 - \Delta & 0 \end{bmatrix} \leq 0$

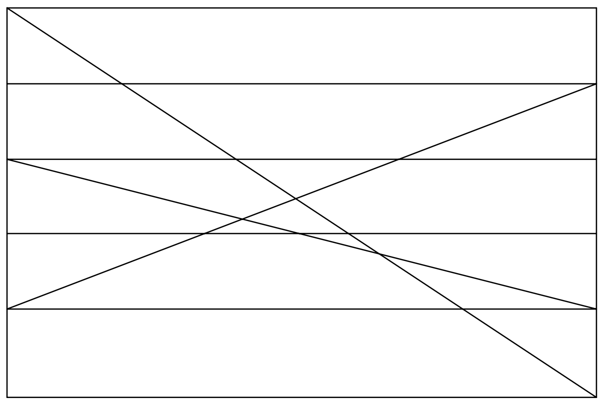
It is also necessary to check

$-\left(c_B + \Delta_{c_B}\right)^T B^{-1} = \begin{bmatrix} -20 & 0 \end{bmatrix} - \begin{bmatrix} \Delta & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} -20 - \Delta & 0 \end{bmatrix} \leq 0$. The decrease allowed is at most 10.

4. Replace the function with $px + 4y + 3z$. Look at the dual problem. Everything is the same as before except the first constraint, which becomes $2\lambda_1 + \lambda_2 \geq p$. The "old" solution $\hat{\lambda} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ is still feasible as long as $p \leq 3$. The profit / item must increase by more than 2 units.

5. (a) $\begin{bmatrix} 2 \\ 5 \end{bmatrix}$ is dominated

(b)



$4x + (1-x) = x + 3(1-x) \Rightarrow x = \frac{2}{5} \Rightarrow v = \frac{11}{5}$ so P_I plays $\begin{bmatrix} \frac{2}{5} \\ \frac{3}{5} \end{bmatrix}$

(c) There is slack in the second constraint. The first line is in a $\frac{6}{5}$ to $\frac{4}{5}$ proportion to the third line in relation to the value line $v = \frac{11}{5}$. It follows that P_{II} plays $\begin{bmatrix} \frac{2}{5} \\ 0 \\ \frac{3}{5} \end{bmatrix}$.

6. Adjust the payoff matrix by adding 2 to each entry.

(a) The primal problem is $\max x + y$ *when* $\begin{cases} 2x + 4y \le 1 \\ 4x + y \le 1 \\ 3x + 2y \le 1 \\ x \ge 0,\, y \ge 0 \end{cases}$

(b) $\hat{x} = \begin{bmatrix} \frac{3}{14} \\ \frac{1}{7} \end{bmatrix}$, $\hat{\lambda} = \begin{bmatrix} \frac{3}{14} \\ \frac{1}{7} \\ 0 \end{bmatrix}$, $\frac{1}{v} = \frac{5}{14}$ so it follows that $p = v\hat{x} = \begin{bmatrix} \frac{3}{5} \\ \frac{2}{5} \end{bmatrix}$ and $q = q = v\hat{\lambda} = \begin{bmatrix} \frac{3}{5} \\ \frac{2}{5} \\ 0 \end{bmatrix}$.

(c) The value before adjustment is given by $\frac{14}{5} - 2 = \frac{4}{5}$ so a fair game is given by $\begin{bmatrix} -\frac{4}{5} & \frac{6}{5} \\ \frac{6}{5} & -\frac{9}{5} \\ \frac{1}{5} & -\frac{4}{5} \end{bmatrix}$