Math 444 Name:_____
Final Exam S. S. #:_____

Each problem is worth 25 points.
Do any six of the first seven problems. Also do problem #8 and #9.

1. Consider the problem:

$$\max\ x_1 - 2x_2 + 3x_3 \text{ when } \begin{cases} 8x_1 + 2x_2 + x_3 \le 12 \\ 2x_1 + x_2 \le 6 \\ -4x_2 + 2x_3 \le 8 \\ x_1, x_2, x_3 \ge 0 \end{cases}.$$

State the dual problem and use the simplex algorithm to solve both the primal and the dual problem.

Make sure that each step employed is the step suggested by the simplex algorithm.

Check your result by evaluating the primal and the dual function.

2. Solve the following problem using both phases of the two-phase method.

$$\max\ x_1 + 2x_2 \text{ when } \begin{cases} x_1 - x_3 \le -6 \\ x_2 + x_3 \le 12 \\ x_1, x_2, x_3 \ge 0 \end{cases}.$$

Make sure that each step employed is the step suggested by the two-phase method.

3. The problem:

$$\max\ 8x_1 + 17x_2 + 7x_3 \text{ when } \begin{cases} x_1 + 2x_2 \le 2 \\ 2x_2 + x_3 \le 2 \\ x_1 + 3x_2 + x_3 \le 3 \\ x_1, x_2, x_3 \ge 0 \end{cases}$$

has a global maximum $\hat{x}_1 = 2, \hat{x}_2 = 0, \hat{x}_3 = 1$.

Use nothing but complementary slackness to find the dual solution $\hat{\lambda}$.

4. Suppose the primal problem:

$$\max c^T x \text{ when } \begin{cases} Ax \leq b \\ x \geq 0 \end{cases}$$

has

$$A = \begin{bmatrix} 1 & 2 \\ 1 & 1 \\ 3 & 2 \end{bmatrix}.$$

The final tableau is given by:

$$\begin{array}{ccccc|c} 0 & 1 & 1 & -1 & 0 & 5 \\ 1 & 0 & -1 & 2 & 0 & 4 \\ 0 & 0 & 1 & -4 & 1 & 2 \\ \hline 0 & 0 & -1 & -1 & 0 & -23 \end{array}.$$

(a) Determine c.

(b) Determine c_B.

5. Suppose $c = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$ yields the final tableau:

$$\begin{array}{cccc|c} 1/2 & 1 & 1/6 & 0 & 3 \\ 7/2 & 0 & -1/6 & 1 & 9 \\ \hline -1/2 & 0 & -1/2 & 0 & -9 \end{array}$$

(a) The first variable is non-basic. How much can $c_1 = 1$ be <u>increased</u> and still have the same basic variables?

(b) The second variable is basic. How much can $c_2 = 3$ be <u>decreased</u> and still have the same basic variables?

6. Suppose $b = \begin{bmatrix} 60 \\ 45 \\ 50 \end{bmatrix}$ yields the final tableau:

$$\begin{array}{ccccccc|c} 0 & 1 & 2/7 & 3/7 & -2/7 & 0 & 90/7 \\ 1 & 0 & 11/7 & -3/7 & 3/7 & 0 & 75/7 \\ 0 & 0 & 8/7 & -8/21 & 3/7 & 1 & 95/7 \\ \hline 0 & 0 & -209/7 & -65/7 & -15/7 & 0 & -4575/7 \end{array}.$$

(a) Determine the largest interval of variation for $b_1 = 60$, with the final tableau still applicable

(b) What is the 'new' maximum value attained by the largest permissible variation of $b_1 = 60$?

7. Consider the problem:

$$\max\ 3x_1 + x_2 + 3x_3 \text{ when } \begin{cases} 2x_1 + x_2 + x_3 \le 2 & \text{(labor time)} \\ x_1 + 2x_2 + 3x_3 \le 5 & \text{(equipment time)} \\ 2x_1 + 2x_2 + x_3 \le 6 & \text{(raw material)} \\ x_i \ge 0,\ i \in \{1,2,3\} \end{cases}.$$

The corresponding final tableau is given by:

1	1/5	0	3/5	−1/5	0	1/5
0	3/5	1	−1/5	2/5	0	8/5
0	1	0	−1	0	1	4
0	−7/5	0	−6/5	−3/5	0	−27/5

Assume the dual solution is unique. A fourth product uses 1 unit of labor time per item, 2 units of equipment time per item, and 3 units of raw material per item.
Find the minimal acceptable profit per item of the fourth product in order to be at all considered as an addition to the product line.

8. Two players in Two-Finger Morra simultaneously put out either one or two fingers. Each player must also announce the number of fingers that he believes his opponent has put out. If neither or both players correctly guess the number of fingers put out by the opponent, the game is a draw. Otherwise, the player who guesses correctly wins (from the other player) the sum (in dollars) of the fingers put out by the two players.

(a) Determine the payoff matrix for Two-Finger Morra. Use the following order of the plans for both players: (show 1, say one), (show 1, say two), (show 2, say one), (show 2, say two),

(b) Setup the primal problem for the column player (who prefers smaller values).

(c) One implementation of the simplex algorithm yields the following bottom row

$\times\ \times\ \times\ \times\ 0\ -\frac{4}{35}\ -\frac{3}{35}\ 0\ -\frac{1}{5}$

Since the game is "symmetric", both players use the same probability distribution. Explain, in terms that a non-specialist can understand, how to play the game "optimally".

(d) What is the expected payoff if the row player (who prefers larger values) decides, against better judgment, to use each plan with the same probability.

9. The function $f : \mathbb{R}^4 \to \mathbb{R}$ given by $f(x_1, x_2, x_3, x_4) = 2\big(x_1 + x_2 + x_3\big) + x_4^2$ is continuous. The four-dimensional ball $C \subset \mathbb{R}^4$ given by $C = \Big\{x = (x_1, x_2, x_3, x_4) \in \mathbb{R}^4 \;\Big|\; x_1^2 + x_2^2 + x_3^2 + x_4^2 \le 1\Big\}$ is closed and bounded. It follows that there is a global maximum of the function $f_0 : C \to \mathbb{R}$ given by $f_0(x) = f(x)$. Find the global maximum of $f_0 : C \to \mathbb{R}$.