6 Conversions.

6.1 Minimization-maximization.

Consider a real-valued function $f : C \to \mathbb{R}$ defined on some set C. Let $g : C \to \mathbb{R}$ be given by $g(x) = -f(x)$. If $\hat{x} \in C$ is a maximizer of f, then $\hat{x}$ is a minimizer of g. To see this, observe that

$$g(\hat{x}) = -f(\hat{x}) \leq -f(x) = g(x)$$

for all $x \in C$.

Exercise

Show that if $\hat{x} \in C$ is a minimizer of $f : C \to \mathbb{R}$, then $\hat{x}$ is a maximizer of $g : C \to \mathbb{R}$, where g is given by $g(x) = -f(x)$.

6.2 Reversal of inequalities.

Inequalities of the form $ax \geq b$ are converted to the standard form by multiplying both sides by -1.

Exercise

First *convert* the standard dual problem to a standard primal problem. Next take the dual of the converted problem. Do you recognize this problem?

6.3 Splitting equalities.

Equalities of the form $Dx = e$ split into a pair $Dx \leq e$ and $Dx \geq e$. In the primal problem this pair appears as

$$\begin{aligned} Dx &\leq e \\ -Dx &\leq -e \end{aligned}.$$

It is of course possible to use the system $Dx = e$ and solve for some of the variables x_i in terms of the others and then replace the x_i throughout the problem.

6.4 Unrestricted variables.

The restriction that all variables are non-negative is possible to circumvent by increasing the number of variables. Suppose that x_i is allowed to be both positive and negative. Introduce $p_i \geq 0$ and $n_i \geq 0$. Replace x_i throughout the problem with $x_i = p_i - n_i$. Observe how the linearity is retained. In this way each converted unrestricted variable increases the dimension by 1.

Exercise

Consider the problem

$$\max\ x_1 + 2x_2 \ \text{ when } \begin{cases} x_1 \leq 3 \\ x_2 \leq 5 \\ x_1 \geq 0 \end{cases}.$$

Convert this problem to a standard primal problem.