

1. Determine the general solution of each of the following differential equations.

(a) $y^{(3)} + 6y'' + 9y' = 0$

(b) $y'' + 2y' + 5y = 0$

(c) $y^{(4)} - 81y = 0$

(d) $y^{(4)} - 2y^{(3)} + 5y'' = 0$

2. Using the method of undetermined coefficients, find the solution of the initial-value problem

$$y'' - 4y' - 5y = 4e^{3x}; \quad y(0) = 2, \quad y'(0) = 8$$

3. Determine the form of a particular solution of the following differential equation. (Do not solve for the unknown coefficients.)

$$y^{(3)} - y'' = 1 + e^{-x} + xe^x.$$

4. Determine the solution of the initial value problem

$$y'' - 5y' + 4y = 2e^{2x}, \quad y(0) = 3, \quad y'(0) = 5$$

5. Use the method of undetermined coefficients to find the general solution of

$$y^{(3)} + 4y'' + 3y' = x$$

6. An object of mass m is dropped at time $t = 0$ and falls under the influence of gravity. Assume that the force of air resistance is proportional to the velocity, and that the acceleration of gravity is a constant $g = 32 \text{ ft/sec}^2$.

(a) Set up and solve the initial value problem to determine the velocity $v(t)$ as a function of t .

(b) If the terminal velocity is -16 ft/sec , what is the velocity at time $t = \ln 2$?

7. Consider the initial value problem $y' = xy$, $y(1) = 2$.

(a) Perform two steps of Euler's method with a step size of $h = 1/2$ to obtain an approximation of $y(2)$.

(b) Perform one step of the improved Euler method with a step size of $h = 1$ to obtain an approximation of $y(2)$.

Answers on the reverse side

These old exam questions are provided in case they help you in studying for the exam. However, our exam may be quite different, so **you should not use this as your only study material**.

ANSWERS:

1. (a) $y(x) = c_1 + c_2e^{-3x} + c_3xe^{-3x}$
(b) $y(x) = c_1e^{-x} \cos 2x + c_2e^{-x} \sin 2x$
(c) $y = c_1e^{3x} + c_2e^{-3x} + c_3 \cos 3x + c_4 \sin 3x.$
(d) $y = c_1 + c_2x + c_3e^x \cos 2x + c_4e^x \sin 2x.$
2. $y(x) = 2e^{5x} + \frac{1}{2}e^{-x} - \frac{1}{2}e^{3x}$
3. $y_p = Ax^2 + Be^{-x} + (Cx^2 + Dx)e^x$
4. $y(x) = e^{4x} + 3e^x - e^{2x}.$
5. $y(x) = c_1 + c_2e^{-x} + c_3e^{-3x} - \frac{4}{9}x + \frac{1}{6}x^2.$
6. (a) DE: $v' = -(g + \rho v)$, where $\rho = k/m$. Solution: $v(t) = \frac{g}{\rho}(e^{-\rho t} - 1).$
(b) -12 ft/sec
7. (a) $21/4$
(b) 7