

NAME & ZID:

SCORE:

1. (15 points) Solve the initial-value problem $y'' = 4y$, $y(0) = 1$, $y'(0) = -14$.

2. (14 points each) Solve each differential equation.

a) $\frac{dy}{dx} = 10e^{5x}(y - 4)^8$.

b) $(\cos x + x^2 e^y + 5y^4) \frac{dy}{dx} + 2x e^y - y \sin x = 0.$

c) $(x - y) \frac{dy}{dx} = 3x - y.$

d) $y^{(4)}(x) - 6y'''(x) - 7y''(x) = 42x.$

e) $y''(x) - 2y'(x) + y(x) = \frac{e^x}{x}.$

3. (10 points) Use Euler's method with step size $h = 1$ to approximate $y(1)$ if

$$\frac{dy}{dx} = x + 4y, \quad y(-2) = 0.$$

4. (20 points) Solve the system $x'(t) = 5x(t) - 2y(t)$, $y'(t) = 4x(t) - y(t)$.

5. (10 points) Compute the convolution $t * e^{3t}$ using the *definition*.

6. (20 points) Solve the initial-value problem $y''(t) + 5y(t) = f(t)$, $y(0) = 0$, $y'(0) = 0$,
where $f(t) = \begin{cases} 4 & \text{if } t < 7, \\ 0 & \text{if } t \geq 7. \end{cases}$

7. (10 points) A mass weighing 60 kilograms is attached to the end of a spring that is stretched 0.05 meters by a force of 507 newtons. An external force in the form $80 \cos(\omega t)$ acts on the mass, where ω is a constant. Find the frequency of the external force in hertz for which resonance occurs.
8. (20 points) A component, initially at 36 degrees, is placed in a large room whose temperature A is kept at 100 degrees. After 2 minutes, the temperature T of the component is found to be 64 degrees. When will the temperature T of the component be 73 degrees? *Hint: Newton's Cooling Law is $dT/dt = k(A - T)$.*

9. (25 points) A 400-liter tank initially contains 100 liters of brine with 52 kilograms of salt in it. Brine containing 1 kilogram of salt per liter enters the tank at the rate of 6 liters per second, and the well-mixed brine in the tank flows out at the rate of 4 liters per second. How much salt will the tank contain when it is just full of brine?