

Much of the material we will cover in this class relies heavily on integration techniques from Calculus II. Use these exercises to refresh your memory and to identify any areas of weakness that you need to reinforce. Answers are provided on the back of this sheet.

I. Simple Substitutions

$$\begin{array}{ll} 1) \int (2 - 3x)^4 dx; & 2) \int x^2 \sqrt{2x^3 - 4} dx \\ 3) \int \frac{[\ln(t)]^{10}}{t} dt; & 4) \int \frac{\cos \theta}{1 + \sin^2 \theta} d\theta \end{array}$$

II. Trigonometric Integrals

$$\begin{array}{ll} 5) \int \sin^3(x) dx & 6) \int \sin^2(x) \cos^3(x) dx \\ 7) \int \sin(3x) \cos(5x) dx & \end{array}$$

III. Integration by Parts

$$\begin{array}{ll} 8) \int x e^{2x} dx & 9) \int t \sin t dt \\ 10) \int \sqrt{y} \cdot \ln(y) dy; & 11) \int e^{3x} \cdot \cos(3x) dx \end{array}$$

IV. Trigonometric Substitution

$$\begin{array}{ll} 12) \int \frac{(1 - x^2)^{1/2}}{x^2} dx; & 13) \int \frac{\sqrt{1 + x^2}}{x^2} dx \\ 14) \int \frac{1}{x} \sqrt{x^2 - 25} dx; & \end{array}$$

V. Quadratic Polynomials

$$15) \int \frac{dx}{x^2 + 4x + 5}; \quad 16) \int x \sqrt{3 - 2x - x^2} dx$$

VI. Rational Functions & Partial Fractions

$$\begin{array}{ll} 17) \int \frac{x^2}{x + 1} dx; & 18) \int \frac{dx}{(x + 1)(x^2 + 1)} \\ 19) \int \frac{2x - 4}{x^2 - x} dx & 20) \int \frac{x + 1}{x^3 - x^2} dx \end{array}$$

Answers

1) $-\frac{1}{15}(2-3x)^5 + c$

2) $\frac{1}{9}(2x^3-4)^{3/2} + c$

3) $\frac{1}{11}[\ln(t)]^{11} + c$

4) $\arctan(\sin \theta) + c$

5) $\frac{1}{3}\cos^3(x) - \cos(x) + c$

6) $\frac{1}{3}\sin^3(x) - \frac{1}{5}\sin^5(x) + c$

7) $-\frac{1}{16}\cos(8x) + \frac{1}{4}\cos(2x) + c$

8) $\frac{1}{2}xe^{2x} - \frac{1}{4}e^{2x} + c$

10) $\frac{2}{3}y^{\frac{3}{2}}\ln(y) - \frac{4}{9}y^{\frac{3}{2}} + c$

9) $-t \cos t + \sin t + c$

11) $\left(\frac{\cos(3x) + \sin(3x)}{6}\right)e^{3x} + c$

12) $-\frac{(1-x^2)^{\frac{1}{2}}}{x} - \arcsin(x) + c$

13) $\ln|x + (1+x^2)^{\frac{1}{2}}| - \frac{(1+x^2)^{\frac{1}{2}}}{x} + c$

14) $(x^2 - 25)^{\frac{1}{2}} - 5\operatorname{arcsec}\left(\left|\frac{x}{5}\right|\right) + c$

15) $\arctan(x+2) + c$

16) $-2\arcsin\left(\frac{x+1}{2}\right) - \frac{1}{2}(x+1)(3-2x-x^2)^{\frac{1}{2}} - \frac{1}{3}(3-2x-x^2)^{\frac{3}{2}} + c$

17) $\frac{1}{2}x^2 - x + \ln(|x+1|) + c$

18) $\frac{1}{2}\ln(|x+1|) - \frac{1}{4}\ln(x^2+1) + \frac{1}{2}\arctan(x) + c$

19) $4\ln|x| - 2\ln|x-1| + c$

20) $-2\ln|x| + \frac{1}{x} + 2\ln(|x-1|) + c$