Math 662, Fall 2011 **Homework 7** **Due: Thurs., Nov. 18**

1. Let $\{\ell_i(x)\}_{i=0}^n$ denote the Lagrange cardinal functions for $n+1$ distinct points $x_0, x_1, \ldots x_n$. Prove that $\sum_{i=0}^{n} \ell_i(x) = 1$ for all x.

2. Text, Problem 6.1.11 (p. 324)

3. Text, Problem 6.1.13. Also, determine a bound on the relative error in the case that we choose the 23 Chebyshev points in $[-1, 1]$ as nodes.

4. Text, Problem 6.1.31.

5. Let $x_0, x_1, \ldots x_n$ be distinct. Prove that $f[x_0, x_1, \ldots x_n] = \sum_{i=0}^{n} w_i f(x_i)$, where $w_i = \prod_{\substack{j=0 \\ j \neq i}}^{n} (x_i - x_j)^{-1}$.

6. Prove that
$$\sum_{i=0}^{n} w_i x_i^n = 1 \text{ and } \sum_{i=0}^{n} w_i = \begin{cases} 1 \text{ if } n = 0 \\ 0 \text{ if } n > 0 \end{cases},$$
where the w_i are defined as in the problem above.

7. Text, Problem 6.2.16.