Math 662, Fall 2011 **Homework 9** **Due: Tues., Nov. 29**

1. (a) Find the best uniform approximation $p_1 \in \Pi_1$ to $f(x) = \ln x$ on $[1, 3]$.
   (b) Find the polynomial $p_1^* \in \Pi_1$ to minimize $\int_1^3 (\ln x - p_1^*)^2 dx$. Compare with the result of part (a) in terms of the uniform norm and in terms of the least-squares norm.
2. Text, Problem 6.9.16 (p. 420)
3. Text, Problem 6.8.11 (p. 404)
4. Use the three-term recurrence relation to find the monic orthogonal polynomials $p_0, p_1, \ldots p_4$ that are orthogonal with respect to the inner product

$$\langle f(x), g(x) \rangle = \int_0^\infty f(x) g(x) e^{-x}\, dx.$$

   (These are called the monic *Laguerre polynomials*.)
5. Let $f(x) \in C[a, b]$, and let $p_n^* \in \Pi_n$ be the polynomial that minimizes $\int_a^b (f(x) - p_n(x))^2 w(x) dx$ over all $p_n \in \Pi_n$, where $w(x)$ is a positive weight function on $[a, b]$. Prove that $p_n^*(x)$ interpolates $f(x)$ at $n + 1$ distinct points in (a, b).
6. Text, Problem 7.2.1 (p. 488)
7. Text, Problem 7.2.13