Review of Rational Expressions and Logarithmic Functions

Author: Dr. Awanou

Most common mistake

$$\frac{1}{x+y} = \frac{1}{x} + \frac{1}{y}, \quad \text{the inequality does not hold,} \quad \text{take x=1,y=1}$$

Other identities

$$a^{-n} = \frac{1}{a^n}$$

$$a^{-r} + a^{-s} \neq a^{-(r+s)}$$

$$a^{-r} + a^{-s} = \frac{1}{a^r} + \frac{1}{a^s} = \frac{a^r + a^s}{a^{r+s}} = (a^r + a^s)a^{-(r+s)}$$

$$\frac{a}{b} + \frac{c}{d} = \frac{ad+bc}{bd}$$

$$\frac{a}{b} + c = \frac{a+bc}{b} \quad (\text{ because } \frac{a}{b} + c = \frac{a}{b} + \frac{cb}{c})$$

$$\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$$

$$\frac{\frac{a}{b}}{c} = \frac{a}{bc} \quad (\text{ because } \frac{\frac{a}{b}}{c} = \frac{\frac{a}{b}}{\frac{c}{1}} = \frac{a}{b} \times \frac{1}{c} = \frac{a}{bc})$$

$$\frac{a}{\frac{b}{c}} = \frac{ac}{b} \quad (\text{because } \frac{a}{\frac{b}{c}} = \frac{\frac{a}{1}}{\frac{b}{c}} = \frac{a}{1} \times \frac{c}{b} = \frac{ac}{b})$$

$$\frac{a}{b} = \frac{a/c}{b/c} \quad (\text{because } \frac{a/c}{b/c} = \frac{a}{c} \times \frac{c}{b} = \frac{a}{b})$$

$$\frac{a^r}{a^s} = a^{r-s} \quad (\text{ e.g. } \frac{a^{3t}}{a^{2t}} = a^t \text{ not } a^{\frac{3}{2}})$$

$$y^{-1} = a + b \text{ gives } y = \frac{1}{a+b}$$

$$\ln(x+y) \neq \ln(x) + \ln(y) \text{ but } \ln(xy) = \ln(x) + \ln(y)$$

$$e^{x+y} = e^x \cdot e^y \text{ but } e^{xy} = (e^x)^y$$