1.2 Primes

Proposition 1.2.2 states that integers a and b are relatively prime if and only if there exist integers m and n with $ma + nb = 1$. This is one of the most useful tools in working with relatively prime integers. Remember that this only works in showing that $\gcd(a, b) = 1$. More generally, if you have a linear combination $ma + nb = d$, it only shows that $\gcd(a, b)$ is a divisor of d (refer back to Theorem 1.1.6).

Since the fundamental theorem of arithmetic (on prime factorization) is proved in this section, you now have some more familiar techniques to use.

SOLVED PROBLEMS: §1.2

23. (a) Use the Euclidean algorithm to find $\gcd(1776, 1492)$.
    (b) Use the prime factorizations of 1492 and 1776 to find $\gcd(1776, 1492)$.
24. (a) Use the Euclidean algorithm to find $\gcd(1274, 1089)$.
    (b) Use the prime factorizations of 1274 and 1089 to find $\gcd(1274, 1089)$.
25. Give the lattice diagram of all divisors of 250. Do the same for 484.
26. Find all integer solutions of the equation $xy + 2y - 3x = 25$.
27. For positive integers a, b, prove that $\gcd(a, b) = 1$ if and only if $\gcd(a^2, b^2) = 1$.
28. Prove that $n - 1$ and $2n - 1$ are relatively prime, for all integers $n > 1$. Is the same true for $2n - 1$ and $3n - 1$?
29. Let m and n be positive integers. Prove that $\gcd(2^m - 1, 2^n - 1) = 1$ if and only if $\gcd(m, n) = 1$.
30. Prove that $\gcd(2n^2 + 4n - 3,\ 2n^2 + 6n - 4) = 1$, for all integers $n > 1$.