

Review Problems

1. For the function $f : \mathbf{R} \rightarrow \mathbf{R}$ defined by $f(x) = x^2$, for all $x \in \mathbf{R}$, describe the equivalence relation on \mathbf{R} that is determined by f .
2. Define $f : \mathbf{R} \rightarrow \mathbf{R}$ by $f(x) = x^3 + 3xz - 5$, for all $x \in \mathbf{R}$. Show that f is a one-to-one function.
Hint: Use the derivative of f to show that f is a strictly increasing function.
3. On the set \mathbf{Q} of rational numbers, define $x \sim y$ if $x - y$ is an integer. Show that \sim is an equivalence relation.
4. In S_{10} , let $\alpha = (1, 3, 5, 7, 9)$, $\beta = (1, 2, 6)$, and $\gamma = (1, 2, 5, 3)$. For $\sigma = \alpha\beta\gamma$, write σ as a product of disjoint cycles, and use this to find its order and its inverse. Is σ even or odd?
5. Define the function $\phi : \mathbf{Z}_{17}^{\times} \rightarrow \mathbf{Z}_{17}^{\times}$ by $\phi(x) = x^{-1}$, for all $x \in \mathbf{Z}_{17}^{\times}$. Is ϕ one to one? Is ϕ onto? If possible, find the inverse function ϕ^{-1} .
6. (a) Let α be a fixed element of S_n . Show that $\phi_{\alpha} : S_n \rightarrow S_n$ defined by $\phi_{\alpha}(\sigma) = \alpha\sigma\alpha^{-1}$, for all $\sigma \in S_n$, is a one-to-one and onto function.
(b) In S_3 , let $\alpha = (1, 2)$. Compute ϕ_{α} .