

ABSTRACT ALGEBRA:

**A STUDY GUIDE
FOR BEGINNERS**

John A. Beachy

Northern Illinois University

2000

This is a supplement to

Abstract Algebra, *Second Edition*
by John A. Beachy and William D. Blair
ISBN 0-88133-866-4, Copyright 1996
Waveland Press, Inc.
P.O. Box 400
Prospect Heights, Illinois 60070
847 / 634-0081
www.waveland.com

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Formatted July 20, 2001, at which time the original was available at:

http://www.math.niu.edu/~beachy/abstract_algebra/

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PREFACE

I first taught an abstract algebra course in 1968, using Herstein's *Topics in Algebra*. It's hard to improve on his book; the subject may have become broader, with applications to computing and other areas, but *Topics* contains the core of any course. Unfortunately, the subject hasn't become any easier, so students meeting abstract algebra still struggle to learn the new concepts, especially since they are probably still learning how to write their own proofs.

This "study guide" is intended to help students who are beginning to learn about abstract algebra. Instead of just expanding the material that is already written down in our textbook, I decided to try to teach by example, by writing out solutions to problems. I've tried to choose problems that would be instructive, and in quite a few cases I've included comments to help the reader see what is really going on. Of course, this study guide isn't a substitute for a good teacher, or for the chance to work together with other students on some hard problems.

Finally, I would like to gratefully acknowledge the support of Northern Illinois University while writing this study guide. As part of the recognition as a "Presidential Teaching Professor," I was given leave in Spring 2000 to work on projects related to teaching.

DeKalb, Illinois
October 2000

John A. Beachy

