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FIELDS

These review problems cover only the first three sections of the chapter. If you are studying abstract algebra because you plan to be a high school teacher, it is precisely these sections (along with the earlier material on polynomials) that are the most relevant to what you will be teaching.

Review Problems

1. Let u be a root of the polynomial x^3+3x+3. In $\mathbf{Q}(u)$, express $(7-2u+u^2)^{-1}$ in the form $a+bu+cu^2$.

2. (a) Show that $\mathbf{Q}(\sqrt{2}+i) = \mathbf{Q}(\sqrt{2}, i)$.
   (b) Find the minimal polynomial of $\sqrt{2}+i$ over $\mathbf{Q}$.

3. Find the minimal polynomial of $1+\sqrt[3]{2}$ over $\mathbf{Q}$.

4. Show that $x^3+6x^2-12x+2$ is irreducible over $\mathbf{Q}$, and remains irreducible over $\mathbf{Q}(\sqrt[5]{2})$.

5. Find a basis for $\mathbf{Q}(\sqrt{5}, \sqrt[3]{5})$ over $\mathbf{Q}$.

6. Show that $[\mathbf{Q}(\sqrt{2}+\sqrt[3]{5}) : \mathbf{Q}] = 6$.

7. Find $[\mathbf{Q}(\sqrt[7]{16} + 3\sqrt[7]{8}) : \mathbf{Q}]$.

8. Find the degree of $\sqrt[3]{2} + i$ over $\mathbf{Q}$. Does $\sqrt[4]{2}$ belong to $\mathbf{Q}(\sqrt[3]{2} + i)$?