

12/13/2002 NO CALCULATORS! Show all necessary work. Be neat, clear, and brief.

1. (30 points) Find the derivative of each of these functions. You do not need to simplify your answers.

(a)  $f(x) = \frac{x^4}{4} - \sqrt[4]{x} - \frac{1}{x}$

(b)  $f(x) = (x^3 - 4x)^2$

(c)  $f(x) = x^2 \ln x - \frac{1}{2x^2}$

(d)  $f(x) = \frac{xe^{-x}}{1+x^2}$

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TOTAL	/ 200
GRADE	

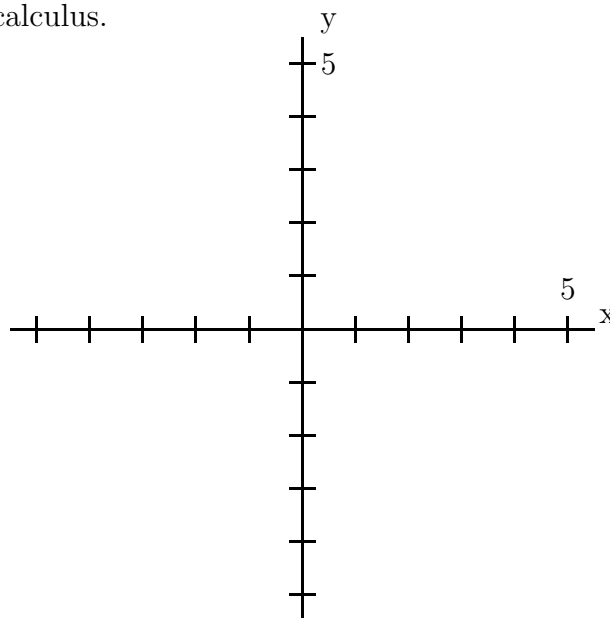
2. (15 pts) Find the following limits algebraically.

(a)  $\lim_{x \rightarrow -5} \frac{x^2 - 5}{x + 5} =$

(b)  $\lim_{x \rightarrow 2} \frac{x^2 + x - 6}{x^2 - 4} =$

3. (20 pts) Find the relative maximum and relative minimum points of  $f(x) = -x^3 + 3x - 2$ .

Graph the function  $f(x)$ , using your knowledge of calculus.



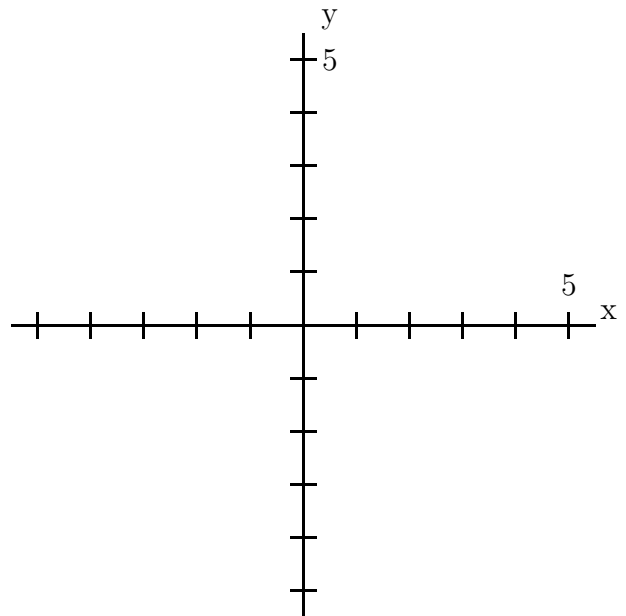
4. (30 pts) Find the following integrals.

(a)  $\int (x^4 - x^{6/5}) dx =$

(b)  $\int_1^e (x + \frac{1}{x}) dx =$

(c)  $\int x^2 \sqrt{x^3 + 1} dx =$

5. (15 pts) Find the area bounded by the curves  $y = x^2$  and  $y = x + 2$ .  
First sketch the graphs of the curves.



6. (15 pts) An appliance firm is marketing a new refrigerator. It determines that in order to sell  $x$  refrigerators, the price per refrigerator must be  $p = 280 - 0.4x$ . It also determines that the total cost of producing  $x$  refrigerators is given by  $C(x) = 5000 + 0.6x^2$ . What price per refrigerator must be charged in order to make a maximum profit?

7. (15 pts) From a thin piece of cardboard 20 in. by 20 in. square corners are cut out so that the sides can be folded up to make a box. What dimensions will yield a box of maximum volume?

8. (15 pts) The population of a colony of bacteria after  $t$  hours is given by  $P(t) = 5000e^{0.02t}$ .
- (a) Find the rate of growth of the population after 50 hours.
  - (b) In how many hours will the population double?
- You may leave your answers in terms of  $e^x$  and  $\ln x$ .*

9. (15 pts) Find  $y''$  if  $y = x\sqrt{1+x^2}$ .

10. (10 pts) A company determines that the marginal cost of producing the  $x$ th unit of a certain product is given by  $C'(x) = x^3 - x$ . Find the function  $C(x)$  that gives the total cost, assuming fixed costs to be \$200 (that is,  $C_0 = 200$ ).

11. (10 pts)  $\frac{d}{dx} \ln \left[ \frac{x^5}{(8x + 5)^2} \right] =$

12. (10 pts) Using the limit definition of the derivative (not the power formula), find the derivative  $f'(x)$  of  $f(x) = \frac{1}{x^2}$ .