MATH 211 HOMEWORK 2 NAME SOLUTIONS

Prof. J. Beachy Monday, 9/16/02 Circle C1(T10) C2(T11) C3(Th11)

1. (10 pts; p 137 #18) For the function $f(x) = x^3$, find $f'(x)$ using the definition on page 128 of the text (show your work). Then find an equation of the tangent line to the graph at the point $(-2, -8)$, at the point $(0, 0)$, and at the point $(4, 64)$.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{(x+h)^3 - x^3}{h} = \lim_{h \to 0} \frac{x^3 + 3x^2h + 3xh^2 + h^3 - x^3}{h}$$

$$= \lim_{h \to 0} \frac{3x^2h + 3xh^2 + h^3}{h} = \lim_{h \to 0} \frac{(h)(3x^2 + 3xh + h^2)}{h} = \lim_{h \to 0} 3x^2 + 3xh + h^2 = 3x^2 + 3x(0) + (0)^2 = 3x^2$$

Remember that the derivative (at a point) gives you the slope of the tangent line (at that point).

$f'(-2) = 3(-2)^2 = 12$ so the tangent line at $(-2, -8)$ is $y = 12(x + 2) - 8$

$f'(0) = 3(0)^2 = 0$ so the tangent line at $(0, 0)$ is $y = 0$

$f'(4) = 3(4)^2 = 48$ so the tangent line at $(4, 64)$ is $y = 48(x - 4) + 64$

2. (10 pts; p 137 #20) For the function $f(x) = \dfrac{2}{x}$, find $f'(x)$ using the definition (show your work). Then find an equation of the tangent line to the graph at the point $(-1, -2)$, at the point $(2, 1)$, and at the point $(10, .2)$.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{1}{h}\left(\frac{2}{x+h} - \frac{2}{x}\right) = \lim_{h \to 0} \frac{1}{h}\left(\frac{(2)(x)}{(x+h)(x)} - \frac{(x+h)(2)}{(x+h)(x)}\right)$$

$$= \lim_{h \to 0} \frac{1}{h}\left(\frac{2x - (2x + 2h)}{(x+h)(x)}\right) = \lim_{h \to 0} \frac{1}{h}\left(\frac{-2h}{(x+h)(x)}\right) = \lim_{h \to 0} \frac{-2}{(x+h)(x)} = \frac{-2}{x^2}$$

$f'(-1) = \dfrac{-2}{(-1)^2} = \dfrac{-2}{1} = -2$ so the tangent line at $(-1, -2)$ is $y = -2(x + 1) - 2$

$f'(2) = \dfrac{-2}{2^2} = \dfrac{-2}{4} = -\dfrac{1}{2}$ so the tangent line at $(2, 1)$ is $y = -\dfrac{1}{2}(x - 2) + 1$

$f'(10) = \dfrac{-2}{10^2} = \dfrac{-2}{100} = -.02$ so the tangent line at $(10, .2)$ is $y = -.02(x - 10) + .2$