

1. (10 pts; p 262 #32) When a theatre owner charges \$3 for admission, there is an average attendance of 100 people. For every 10 cent increase in admission, there is a loss of 1 customer from the average number. What admission should be charged in order to maximize revenue?

We need to maximize the revenue.

Let x be the price of a ticket, and let y be the number of tickets sold when the price is x dollars. The total amount of money taken in would be the number of tickets sold multiplied by the price per ticket, so the revenue is yx .

The problem is to find the relationship between y and x . The problem states that if $x = 3$, then $y = 100$. For each increase of 10 cents in the ticket price, the number of tickets goes down by one. This describes a linear function, so we can say that $y = m(x - a) + b$. If we add 1 dollar to the ticket price to get $x = 4$, then y will go down by 10 to 90, and this gives us a second point to use in finding the equation for y .

Revenue: $R(x) = yx$

To find $y = m(x - a) + b$, we have $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{90 - 100}{4 - 3} = -10$. Letting $a = 3$ and $b = 100$,

$$y = -10(x - 3) + 100 = -10x + 130$$

Now substitute into the formula for $R(x)$, find $R'(x)$ and set $R'(x) = 0$.

$$R(x) = (-10x + 130)x = -10x^2 + 130x \quad R'(x) = -20x + 130 \quad 20x = 130 \quad x = 6.5$$

The graph of $R(x)$ is a parabola opening down, so $x = 6.5$ gives an absolute maximum.

Answer: To maximize revenue, charge \$6.50 per ticket.

2. (10 pts; p 262 #52) A power line is to be constructed from a power station to an island. The point on the island is 1 mile from shore, directly opposite a point 4 miles downshore from the power station. It costs \$5000 per mile to lay the power line under water, and \$3000 per mile to lay the line under ground. At what point should the line come to the shore in order to minimize the total cost?

We need to minimize the cost function.

Using the diagram in the text, let x be the distance to the point at which the power line will reach land. Then the distance under water is $\sqrt{1 + x^2}$, giving a cost of $5000\sqrt{1 + x^2}$ for this part of the line. (*Remember: If the sides of a right triangle have lengths a and b and the hypotenuse has length c , the Pythagorean theorem says that $c^2 = a^2 + b^2$.)* The remainder of the line is on land, so the remaining length costs $3000(4 - x)$.

Cost function: $C(x) = 5000\sqrt{1 + x^2} + 3000(4 - x) = 5000\sqrt{1 + x^2} + 12000 - 3000x$

Now we need to find $C'(x)$ and set it equal to zero to find the critical points of the function.

$$C(x) = 5000(1 + x^2)^{1/2} + 12000 - 3000x$$

$$C'(x) = 5000(1/2)(1 + x^2)^{-1/2}(2x) - 3000 = \frac{5000x}{\sqrt{1 + x^2}} - 3000$$

$$\text{Set } C'(x) = 0 \quad \frac{5000x}{\sqrt{1 + x^2}} - 3000 = 0 \quad \frac{5000x}{\sqrt{1 + x^2}} = 3000$$

$$\frac{5}{3}x = \sqrt{1 + x^2} \quad \frac{25}{9}x^2 = 1 + x^2 \quad \frac{16}{9}x^2 = 1 \quad x^2 = \frac{9}{16} \quad x = \frac{3}{4}$$

Using the first derivative test, it is easy to check that $C'(0) = -3000$ is negative, while $C'(1) = -3000 + \frac{5000}{\sqrt{2}}$ is positive. We see that $C(x)$ is decreasing until $x = \frac{3}{4}$, and then increasing after $x = \frac{3}{4}$, so $x = \frac{3}{4}$ gives a local minimum.

We are looking for an absolute minimum on the interval $[0, 4]$. We can tell from the sign of $C'(x)$ that the cost will be larger for $x = 0$ or $x = 4$, so the final answer is $x = \frac{3}{4}$.