MATH 211 FINAL EXAM 12/9/97 NO CALCULATORS!

1. (40 pts.) Find the derivatives.

   (a) $y = 2\sqrt{x} - \dfrac{1}{x^2} + 5x + 10$

   (b) $y = \dfrac{(5x+3)^7}{x^2+1}$

   (c) $y = xe^{x^2} + \sqrt{\ln x}$

   (d) $y = \ln \dfrac{e^x}{1+x^2}$

2. (20 pts.) Find the limits.

   (a) $\displaystyle\lim_{x\to 1} \frac{x^2+x-2}{x^3-x}$

   (b) $\displaystyle\lim_{x\to\infty} \frac{1-2x}{x^2+5}$

3. (15 pts.) Given $f(x) = x^3 - 6x^2$.

   (a) Find all relative extreme points and inflection points.

   (b) Graph the function.

4. (10 pts.) Find an equation of the tangent line to the curve $y = \ln(2x+1)$ at $x = 1$.

5. (15 pts.) Use the limit definition of derivatives (not the power rule) to find the derivative of $y = \dfrac{1}{x+1}$.

6. (15 pts.) A closed box with a square base is to have a volume of 250 cubic meters. The material for the top and bottom of the box costs \$2 per square meter, and the material for the sides costs \$1 per square meter. Find the dimension of the box for which the cost of materials is minimized.

7. (15 pts.) The demand equation for a product is $p = 750 - \frac{1}{2}x$, where p is the price and x is the level of production. The cost function is $c(x) = 1000 + 150x$. Find the value of x that will maximize the profit.

8. (15 pts.) A radioactive substance decays exponentially. If 100 grams of the substance decays to 50 grams in 3 years, how many grams of the substance will remain after 5 years.

9. (10 pts.) Given $f(x) = \left(\dfrac{1}{e}\right)^x$ and $g(x) = \left(\dfrac{1}{e}\right)^{-x}$, determine which function is increasing and which is decreasing. Why?

10. (30 pts.) Find the following integrals.

    (a) $\displaystyle\int (\sqrt{x} + \frac{3}{x} - 2)dx$

    (b) $\displaystyle\int (e^{3x} + \frac{x^2+x}{x})dx$

    (c) $\displaystyle\int_1^2 \frac{\ln x}{x} dx$

11. (15 pts.). Given $y = x^2$ and $y = x$, find the area of the region between the curves.