MATH 211 FINAL EXAM 12/9/98 NO CALCULATORS!

1. (32 points) Find the derivative of each of the following functions. You do not need to simplify your answers.

(a) $f(x) = \dfrac{x^5}{7x^3 + 6x - 1}$ (b) $f(x) = \ln(5 - 6x^7)$

(c) $f(x) = 4e^{-3x^2+1} + 7$ (d) $f(x) = (3 - 8x)(2x + 7)^5$

2. (14 pts) Find the following limits or show that no limit exists.

(a) $\lim\limits_{x\to 2} \dfrac{x^2 + x - 6}{x - 2} =$ (b) $\lim\limits_{x\to 5} \dfrac{x^3 - 25x}{x^2 - 10x + 25} =$

3. (10 pts) Find the equation of the line tangent to the curve $y = x^2 \ln x$ at the point (e, e^2).

4. (10 pts) Let $f(x) = x^2e^x - 2xe^x$. Find the x-coordinates of all relative extrema of f. Determine whether each extremum is a relative maximum or a relative minimum.

5. (20 pts) Let $f(x) = -x^3 + 3x^2 + 9x - 20$.

a) Find all relative extrema of f (BOTH COORDINATES). Determine whether each extremum is a relative maximum or a relative minimum.
b) Find all inflection points of f (BOTH COORDINATES).
c) Using your knowledge of calculus, sketch a graph of f that includes all the points found in parts a) and b).

6. (24 pts) Compute the following integrals.

(a) $\displaystyle\int_1^2 \left(6x^2 - \frac{3}{4x^2}\right) dx$ (b) $\displaystyle\int \left(\frac{3}{x - 4} - 5\right) dx$ (c) $\displaystyle\int t^2 e^{t^3} dt$

7. (15 pts) Find the area of the region bounded by the curves $y = x^2 + 1$ and $y = -x^2 + 19$.

8. (14 pts) The demand equation for a certain product is $p = 2x + 4$, where p is the price and x is the number of units produced. If the cost function is $C(x) = 3x^2 + 12$, determine the level of production that will maximize the profit.

9. (14 pts) A closed rectangular box with square base and a volume of 12 cubic feet is to be constructed using two differen types of materials. The top is made of a metal costing \$2 per square foot and the remainder of wood costing \$1 per square foot. Find the dimensions of the box for which the cost of materials is minimized.

10. (14 pts) The population of a colony of bacteria doubles every 5 hours.

(a) Find the growth constant for the population.

(b) How long will it take an initial population of 500 bacteria to grow to 10,000?

11. (10 pts) Estimate the area under the graph of the function $f(x) = x^2$ from 0 to 6 using a Riemann sum with $n = 3$ and taking the x_i to be the midpoints of the subintervals.

12. (10 pts) Suppose that $f(3) = 7$ and $f'(3) = -2$. Use the tangent line approximation to estimate $f(3.05)$.

13. (10 pts) Using the limit definition of the derivative (not the power rule), find $f'(x)$ if $f(x) = x^2$.