

Answers to some max-min practice problems:

1. A photographer has a thin piece of wood 16 inches long. How should he cut the wood to make a rectangular picture frame that encloses the maximum area?

Let x =width, y =height. Then $2x + 2y = 16$, so the area is $A = xy = x(8 - x)$. Answer: $x = 4$

2. A rancher has a total of 12 miles of fencing with which to enclose a rectangular pasture. He plans to fence the entire area and then to subdivide it by running a fence across the width. What dimensions should the pasture have in order to enclose the maximum area with the available fencing?

Let x =width, y =length. Then $2x + 3y = 12$, so the area is $A = xy = x(4 - \frac{2}{3}x)$. Answer: $x = 3$, $y = 2$

3. Find the area of the largest rectangle with lower base on the x -axis and upper vertices on the curve $y = 6 - x^2$.

$A = xy = x(6 - x^2) = 6x - x^3$ Answer: $x = \sqrt{2}$

4. A retailer knows that if he charges x dollars for an alarm clock, he will be able to sell $480 - 40x$ clocks. The alarm clock costs him \$4 per clock. How much should he charge per clock in order to maximize his total profit?

$R(x) = x(480 - 40x)$ and $C(x) = 4(480 - 40x)$ so $P(x) = 640x - 40x^2 - 1920$ and the maximum profit occurs at \$8.00 per clock.

5. Referring to the previous problem: If the government adds a tax of \$1 to the price of each clock, how much should the retailer now charge in order to maximize profit?

$R(x) = x(480 - 40x)$ and $C(x) = 5(480 - 40x)$ so $P(x) = 680x - 40x^2 - 2400$ and the maximum profit occurs at \$8.50 per clock. Note that the retailer maximizes his profit by passing on only half of the tax increase.

6. A rectangular trough is formed from a piece of metal 12 inches wide and of unknown length. What should the dimensions be to maximize the amount of water carried?

Maximize the cross-section area. Let x =width, y =height of the trough. Then $x + 2y = 12$, and the area is $A = xy = x(6 - \frac{1}{2}x)$. Answer: $x = 6$

7. You wish to construct a tomato can out of 100π square inches of metal. What is the volume of the largest can that you can construct? (Hint: use x for the radius of the can, and y for its height. Find the volume in terms of these variables.)

Maximize volume= $\pi x^2 y$ (The volume equals the area of the base, which is a circle, times the height.) The top and bottom are circular, and have area πx^2 . The other surface of the can is rectangular, with height y and length $2\pi x$, the circumference of the circular base. (Think of taking a can opener, cutting out the top and bottom, which are circular, and then slicing the remaining cylinder from top to bottom and flattening it out into a rectangle.) Since the surface area is 100π , we have $100\pi = 2\pi x^2 + 2\pi xy$. Solving for y gives $y = \frac{50}{x} - x$. Then $V = \pi x^2 y = 50\pi x - \pi x^3$ is maximum when $x = \sqrt{\frac{50}{3}}$ and $y = 2\sqrt{\frac{50}{3}}$, so the volume is maximum when the height is equal to the diameter of the can.

8. You wish to construct a beer can (as in the previous problem). This means that the top and bottom must be double thickness. What is the largest can that may be constructed out of 100π square inches of metal?

$V = \pi x^2 y$ as before, but now $y = \frac{50}{x} - 2x$, and so $V = 50\pi x - 2\pi x^3$. Answer: $x = \sqrt{\frac{25}{3}}$ and $y = 4\sqrt{\frac{25}{3}}$, so now the height should be twice the diameter.