MATH 211	QUIZ 6	NAME SOLUTIONS
Prof. J. Beachy	Friday, 10/11/02	Circle C1(T10) C2(T11) C3(Th11)

1. (10 pts; p 199 #7) For the function $f(x) = x^3 - x^2 - x + 2$, find $f'(x)$ and $f''(x)$. Find the values of x for which $f'(x)$ is positive; negative; zero. Find the values of x for which $f''(x)$ is positive; negative; zero.

$f'(x) = 3x^2 - 2x - 1 \qquad f''(x) = 6x - 2$

There are two ways to check the sign of $f'(x)$. In either case we should factor, set $f'(x) = 0$ and solve.

$f'(x) = (3x+1)(x-1)$ Set $f'(x) = 0$ $(3x+1)(x-1) = 0$ $3x + 1 = 0$ or $x - 1 = 0$ $x = -1/3$ or $x = 1$

First method: *Analyze the sign of each factor. We can see that the first factor $3x + 1$ of $f'(x)$ changes from negative to positive at $x = -1/3$, and the second factor $x - 1$ changes from negative to positive at $x = 1$.*

For $x < -1/3$, $f'(x)$ is $(neg)(neg) = pos$, so $f(x)$ is increasing.
For $x = -1/3$, $f'(x) = 0$.
For $-1/3 < x < 1$, $f'(x)$ is $(pos)(neg) = neg$, so $f(x)$ is decreasing.
For $x = 1$, $f'(x) = 0$.
For $1 < x$, $f'(x)$ is $(pos)(pos) = pos$, so $f(x)$ is increasing.

Second method: *Use a table, as the author does on page 193. The intervals are chosen by going from one solution of $f'(x) = 0$ to the next, moving from left to right on the number line. In each interval, choose a test value.*

Interval	$(-\infty, -1/3)$	$(-1/3, 1)$	$(1, +\infty)$
Test value	$x = -1$	$x = 0$	$x = 2$
Sign of $f'(x)$	$f'(-1) = (-2)(-2) > 0$	$f'(0) = (1)(-1) < 0$	$f'(2) = (7)(1) > 0$
Result	f is increasing	f is decreasing	f is increasing

Note that I used the factored form $f'(x) = (3x+1)(x-1)$ of the derivative to find $f'(-1)$, $f'(0)$, and $f'(2)$.

To find the values of x for which $f''(x)$ is positive, negative, or zero, since we have a linear function we can just write $f''(x) = 2(3x - 1)$. Now we can see that $f''(x)$ changes sign when $3x - 1 = 0$, which gives us $x = 1/3$.

$f''(x) < 0$ for $x < 1/3$; $f''(x) = 0$ for $x = 1/3$; $f''(x) > 0$ for $1/3 < x$.

2. (10 pts; p 215 #25) For the function, $f(x) = \dfrac{x}{x^2+1}$, find $f'(x)$, and the values of x for which $f'(x)$ is positive; negative; zero. Find the maximum and minimum values of $f(x)$.

We need to use the quotient rule, then simplify and factor.

$$f'(x) = \frac{(1)(x^2+1) - (x)(2x)}{(x^2+1)^2} = \frac{x^2 + 1 - 2x^2}{(x^2+1)^2} = \frac{-x^2+1}{(x^2+1)^2} = \frac{-(x^2-1)}{(x^2+1)^2} = \frac{-(x+1)(x-1)}{(x^2+1)^2}$$

Set $f'(x) = 0$ $\dfrac{-(x+1)(x-1)}{(x^2+1)^2} = 0$ $-(x+1)(x-1) = 0$ $x + 1 = 0$ or $x - 1 = 0$ $x = -1$ or $x = 1$

First method: *Looking at $f'(x)$ in factored form, we can see that the denominator is always positive, so we just need to analyze the sign of the numerator. The sign of the factor $x + 1$ changes at $x = -1$; the sign of the factor $x - 1$ changes at $x = 1$.*

For $x < -1$, $f'(x)$ is $-(neg)(neg)/(pos) = neg$, so $f(x)$ is decreasing.
For $x = -1$, $f'(x) = 0$.
For $-1 < x < 1$, $f'(x)$ is $-(pos)(neg)/(pos) = pos$, so $f(x)$ is increasing.
For $x = 1$, $f'(x) = 0$.
For $1 < x$, $f'(x)$ is $-(pos)(pos)/(pos) = neg$, so $f(x)$ is decreasing.

Second method: *Use a table.*

Interval	$(-\infty, -1)$	$(-1, 1)$	$(1, +\infty)$
Test value	$x = -2$	$x = 0$	$x = 2$
Sign of $f'(x)$	$f'(-2) = -(-1)(-3)/(25) < 0$	$f'(0) = -(1)(-1)/(1) > 0$	$f'(2) = -(3)(1)/(25) < 0$
Result	f is decreasing	f is increasing	f is decreasing

The final answer: The critical points of $f(x)$ are $x = -1$ and $x = 1$.
Since $f(x)$ changes from decreasing to increasing at $x = -1$, the value $f(-1) = -1/2$ is a relative minimum value.
Since $f(x)$ changes from increasing to decreasing at $x = 1$, the value $f(1) = 1/2$ is a relative maximum value.