MATH 211 FINAL EXAM NAME ________________________________

5/6/2003 NO CALCULATORS! Show all necessary work. Be neat, clear, and brief.

1. (40 points) Find the derivative of each of these functions. You do not need to simplify your answers.

(a) $f(x) = 4\sqrt{x} + x - \dfrac{5}{x}$

(b) $f(x) = (\ln x)^2$

(c) $f(x) = \dfrac{e^x}{x^2 + 1}$

(d) $f(x) = \ln\left[\sqrt{5 + x^2}\,\right]$

page 1	/ 40
page 2	/ 45
page 3	/ 25
page 4	/ 30
page 5	/ 35
page 6	/ 25
TOTAL	/ 200
GRADE	

2. (30 pts) Find the following integrals.

(a) $\int \left(x^3 + \frac{1}{x^3} - x^{8/7} \right) \, dx =$

(b) $\int (1-t)\sqrt{t} \, dt =$

(c) $\int_0^1 2xe^{x^2} \, dx =$

3. (15 pts) Find the following limits algebraically.

(a) $\lim_{x \to 4} \frac{x^2 - 16}{x^2 - x - 12} =$

(b) $\lim_{x \to -1} \frac{4x^2 + 5x - 7}{3x^2 - 2x + 1} =$

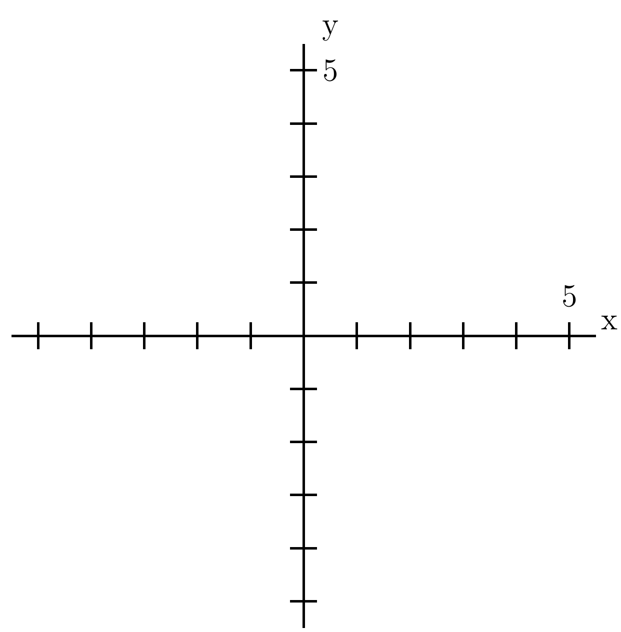
4. (25 pts) Let $f(x) = x + \dfrac{4}{x}$.

(a) Find $f'(x)$ and $f''(x)$.

(b) Find the critical points and undefined values of $f(x)$.

(c) Determine where $f(x)$ is increasing and/or decreasing; find the relative maximum and minimum values of $f(x)$ (use the second derivative test to check your answers).

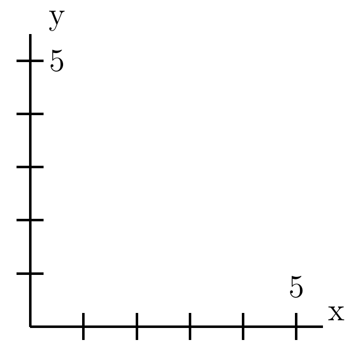
(d) Graph the function $f(x)$, using your knowledge of calculus.



5. (15 pts) A firm estimates that its daily total cost function (in suitable units) is $C(x) = x^3 - 6x^2 + 13x + 15$, and its total revenue function is $R(x) = 28x$.
Find the value of x that maximizes the daily profit.

6. (15 pts) TWA requires that the total dimensions (length + width + height) of a checked bag cannot exceed 62 inches. Suppose that you want to check a bag whose height equals its width. What are the dimensions of the largest bag of this shape that you can check on a TWA flight? (Maximize the volume)

7. (15 pts) Find the area bounded by the curve $y = \frac{1}{x}$ and the lines $y = x$, $x = 1$, and $x = e$. First sketch the graphs.



8. (10 pts) A company finds that the rate at which consumer-demand quantity changes with respect to price is given by the marginal-demand function $D'(p) = -\frac{4000}{p^2}$. Find the demand function if it is known that 1003 units of the product are demanded by consumers when the price is \$4 per unit.

9. (10 pts) Find $f'(x)$ and $f''(x)$ for the function $f(x) = x^2 \ln x$.

10. (15 pts) The power supply of a satellite is a radioisotope. The power output P, in watts, decreases at a rate proportional to the amount present; P is given by $P(t) = 50e^{-0.004t}$, where t is the time, in days. *You may leave your answers in terms of* e^x *and/or* $\ln x$.

(a) How much power is available after 250 days?

(b) What is the half-life of the power supply?

11. (10 pts) Using the limit definition of the derivative (not the power formula), find the derivative $f'(x)$ of $f(x) = \dfrac{3}{x}$.