Math 211 Section C, QUIZ 4 Solutions Friday, 2/7/03

1. (5 pts; p155 #4) Given the revenue function $R(x) = 50x - 0.5x^2$ and cost function $C(x) = 4x + 10$, find the profit function $P(x)$ and the function that gives the marginal profit.

$P(x) = R(x) - C(x) = (50x - 0.5x^2) - (4x + 10) = 46x - 0.5x^2 + 10$ Marginal profit $= P'(x) = 46 - x$.

2. (5 pts; p163 #12) Use the product rule to find $f'(x)$ for $f(x) = (\sqrt[3]{x} - 5x^2 + 4)(4x^2 + 11x - 5)$. (Do not simplify your answer.) First simplify $f(x)$: $f(x) = (x^{1/3} - 5x^2 + 4)(4x^2 + 11x - 5)$

$f'(x) = (\frac{1}{3}x^{-2/3} - 10x)(4x^2 + 11x - 5) + (x^{1/3} - 5x^2 + 4)(8x + 11)$

3. (5 pts; p165 #34) Find and simplify

$$\frac{d}{dx}\left(\frac{3x^2 - 5x}{x^2 - 1}\right) = \frac{(6x - 5)(x^2 - 1) - (3x^2 - 5x)(2x)}{(x^2 - 1)^2} = \frac{6x^3 - 6x - 5x^2 + 5 - 6x^3 + 10x^2}{(x^2 - 1)^2}$$
$$= \frac{5x^2 - 6x + 5}{(x^2 - 1)^2}$$

4. (5 pts; p163 #77) Find an equation of the tangent line to the graph of $y = \dfrac{8}{x^2 + 4}$ at the point $(-2, 1)$.

The derivative is $y' = \dfrac{(0)(x^2 + 4) - (8)(2x)}{(x^2 + 4)^2} = \dfrac{-16x}{(x^2 + 4)^2}$.

Substitute $x = -2$ to get the slope of the tangent line at $(-2, 1)$: $\dfrac{-16(-2)}{((-2)^2 + 4)^2} = \dfrac{32}{8^2} = \dfrac{32}{64} = \dfrac{1}{2}$.

Use $y = m(x - a) + b$ to get the tangent line: $y = \frac{1}{2}(x + 2) + 1$

Math 211 Section C, HOMEWORK 2 Solutions Wednesday, 2/12/03

Note: You do not need to simplify your answers in #1 and #2.

1. (5 pts; p165 #99) Use the product rule and quotient rule to find $f'(t)$ for $f(t) = (t^5 + 3) \cdot \dfrac{t^3 - 1}{t^3 + 1}$.

$f'(t) = (5t^4) \cdot \dfrac{t^3 - 1}{t^3 + 1} \; + \; (t^5 + 3) \cdot \dfrac{(3t^2)(t^3 + 1) - (t^3 - 1)(3t^2)}{(t^3 + 1)^2}$

2. (5 pts; p172 #20) Use the product rule and extended power rule to find $f'(x)$ for $f(x) = (x-4)^8(x+3)^9$.

$f(x) = 8(x - 4)^7(1)(x + 3)^9 \; + \; (x - 4)^8(9)(x + 3)^8(1)$.

3. (5 pts; p172 #47) Find an equation of the tangent line to the graph of $y = \sqrt{x^2 + 3x}$ at the point $(1, 2)$.

To find the slope at x we need to find y'. We can rewrite the function in the form $(x^2 + 3x)^{1/2}$, and then $y' = \frac{1}{2}(x^2 + 3x)^{-1/2}(2x + 3) = \dfrac{2x + 3}{2\sqrt{x^2 + 3x}}$.

When $x = 1$, we have $y' = \dfrac{2(1) + 3}{2\sqrt{(1)^2 + 3(1)}} = \dfrac{5}{2\sqrt{4}} = \dfrac{5}{4}$.

Using the point-slope form of a line, $y = \dfrac{5}{4}(x - 1) + 2$.

4. (5 pts; p173 #68) Find the marginal utility for the utility function $U(x) = 80\sqrt{\dfrac{2x + 1}{3x + 4}}$.

We need to find the derivative $U'(x)$. We first simplify $U(x)$.

$U(x) = 80\sqrt{\dfrac{2x + 1}{3x + 4}} = 80\dfrac{\sqrt{2x + 1}}{\sqrt{3x + 4}} = 80(2x + 1)^{1/2}(3x + 4)^{-1/2}$.

$U'(x) = 80(\frac{1}{2})(2x + 1)^{-1/2}(2)(3x + 4)^{-1/2} \; + \; 80(2x + 1)^{1/2}(-\frac{1}{2})(3x + 4)^{-3/2}(3)$