QUIZ 5 Solutions MATH 211 C

Let $f(x) = x^3 - 3x + 6$. (p 199 #9)

(a) [6 pts] Find $f'(x)$ and the critical points of $f(x)$.

$f'(x) = 3x^2 - 3$ Set $f'(x) = 0$ to get $3x^2 - 3 = 0$ $3x^2 = 3$ $x^2 = 1$ $x = \pm 1$

(b) [4 pts] Find the intervals on which $f(x)$ is increasing and decreasing.

The critical points separate the number line into the intervals $(\infty, -1)$, $(-1, 1)$, and $(1, \infty)$. We need to check the sign of the first derivative at one point in each of the intervals. You could choose -2, 0, and 2 as the points. Then find the value of the derivative at each point.

$f'(-2) = 3(-2)^2 - 3 = +9$ $f'(0) = -3$ $f'(2) = 3(2)^2 - 3 = +9$

Conclusion: $f'(x)$ is positive when x is in the interval $(\infty, -1)$, negative when x is in $(-1, 1)$, and positive when x is in $(1, \infty)$. Therefore $f(x)$ is increasing when x is in the interval $(\infty, -1)$, decreasing when x is in $(-1, 1)$, and increasing when x is in $(1, \infty)$.

(c) [4 pts] Find the relative maximum and relative minimum values of $f(x)$.

Since $f(x)$ increases as you approach $x = -1$ and then decreases afterwards, we can see that $f(-1) = (-1)^3 - 3(-1) + 6 = -1 + 3 + 6 = 8$ must be a relative maximum value.

Since $f(x)$ decreases as you approach $x = 1$ and then increases afterwards, we can see that $f(1) = (1)^3 - 3(1) + 6 = 1 - 3 + 6 = 4$ must be a relative maximum value.

(d) [6 pts] Sketch the graph. There is a link to the graph on the class web page.

QUIZ 6 Solutions

1. (5 pts; p232 #10) $$\lim_{x\to\infty} \frac{4 - 3x - 12x^2}{1 + 5x + 3x^2} = \lim_{x\to\infty} \frac{\frac{4}{x^2} - \frac{3}{x} - 12}{\frac{1}{x^2} + \frac{5}{x} + 3} = \frac{0 - 0 - 12}{0 + 0 + 3} = \frac{-12}{3} = -4$$

2. (20 pts; p215 #28) Let $f(x) = \dfrac{-4}{x^2 + 1}$.

(a) [3 pts] Find all vertical and horizontal asymptotes.

To find vertical asymptotes, set the denominator equal to zero. Since $x^2 + 1 = 0$ has no solution, there are none. To find horizontal asymptotes, find $\lim_{x\to\infty} f(x)$. You should get $y = 0$ as a horizontal asymptote.

(b) [4 pts] Find $f'(x)$ and the critical points of $f(x)$; find where $f(x)$ is increasing and decreasing.

Using the quotient rule, $f'(x) = \dfrac{(0)(x^2+1) - (-4)(2x)}{(x^2+1)^2} = \dfrac{8x}{(x^2+1)^2}$, and then setting $f'(x) = 0$ gives $8x = 0$, since a fraction can only be equal to zero when its numerator is zero. The sign of $f'(x)$ just depends on the sign of $8x$, since the denominator is always positive. This shows that $f'(x)$ is negative when $x < 0$, and positive when $x > 0$, so $f(x)$ is decreasing when $x < 0$, and increasing when $x > 0$.

(c) [4 pts] Find $f''(x)$, and the intervals on which $f(x)$ is concave up and concave down.

$$f''(x) = \frac{(8)(x^2+1)^2 - (8x)(2)(x^2+1)(2x)}{((x^2+1)^2)^2} = \frac{(8)(x^2+1)[(x^2+1) - (x)(2)(2x)]}{(x^2+1)^4} = \frac{(8)(1-3x^2)}{(x^2+1)^3}$$

Since the denominator is always positive, the second derivative can only change sign when the numerator is zero. Set the numerator equal to zero, then test the sign of $f''(x)$ in the corresponding intervals.

$1 - 3x^2 = 0$ $x^2 = \dfrac{1}{3}$ $x = \pm\sqrt{\frac{1}{3}}$ Intervals: $(-\infty, -\sqrt{\frac{1}{3}})$, $(-\sqrt{\frac{1}{3}}, \sqrt{\frac{1}{3}})$, $(\sqrt{\frac{1}{3}}, \infty)$

$f''(-1) = \dfrac{(8)(1 - 3(-1)^2)}{((-1)^2+1)^3} = \dfrac{(8)(1-3)}{(1+1)^3} < 0$ $f''(0) = \dfrac{(8)(1-0)}{(0+1)^3} > 0$ $f''(1) = \dfrac{(8)(1-3)}{(1+1)^3} < 0$

Thus $f(x)$ is concave down on $(-\infty, -\sqrt{\frac{1}{3}})$, concave up on $(-\sqrt{\frac{1}{3}}, \sqrt{\frac{1}{3}})$, and concave down on $(\sqrt{\frac{1}{3}}, \infty)$.

(c) [4 pts] Find the relative maximum and relative minimum values of $f(x)$.

The only critical point is $x = 0$. Since $f(x)$ decreases till $x = 0$ and then increases, it must give a relative minimum. You can also see this from the fact that $f(x)$ is concave up when $x = 0$. The minimum is $f(0) = -4/1 = -4$.

(d) [5 pts] There is a link to the graph on the class web page.