MATH 211, Section C TEST I SOLUTIONS
Prof. J. Beachy 2/14/03

1. (20 pts; 5 pts each) Find the following derivatives, using the formulas we have studied.

(a) $f(x) = x^4 - 5x^3 + 2x + 11$ $\qquad f'(x) = 4x^3 - 15x^2 + 2$

(b) $f(x) = 10\sqrt[5]{x} + 4x^{3/4} - \dfrac{2}{x^4} = 10x^{1/5]} + 4x^{3/4} - 2x^{-4}$ $\qquad f'(x) = 2x^{-4/5} + 3x^{-1/4} + 8x^{-5}$

(c) $f(x) = (4\sqrt{x} - 6)(x^3 - 1)^7 = (4x^{1/2} - 6)(x^3 - 1)^7$
$f'(x) = (2x^{-1/2})(x^3 - 1)^7 + (4x^{1/2} - 6)(7)(x^3 - 1)^6(3x^2)$

(d) $\dfrac{d}{dx}\left(\dfrac{3x^4 - 5x}{x^2 - 1}\right) = \dfrac{(12x^3 - 5)(x^2 - 1) - (3x^4 - 5x)(2x)}{(x^2 - 1)^2}$

2. (5 pts) Find the points on the graph of $y = x^3 - 3x + 2$ at which the tangent line is horizontal.

The tangent line is horizontal when the slope is zero, so we need to set $y' = 0$.

$3x^2 - 3 = 0 \qquad 3x^2 = 3 \qquad x^2 = 1 \qquad x = \pm 1$ $\qquad$ The points are $(-1, 4)$ and $(1, 0)$.

3. (10 pts) Graph $y = \dfrac{2}{x - 1}$. $\qquad$ The shape is almost the same as $y = \dfrac{1}{x - 1}$, on Quiz 1.
(There is a graph posted separately on the web.)

4. (15 pts; compare p163 #77) Find an equation of the tangent line to the graph of $y = \dfrac{2}{x - 1}$ at the point $(0, -2)$. Repeat this for the point $(3, 1)$. Graph the tangent lines on the axes given above.

$y' = \dfrac{(0)(x - 1) - (2)(1)}{(x - 1)^2} = \dfrac{-2}{(x - 1)^2}$ (quotient rule). When $x = 0$, $y' = -1$, and when $x = 3$, $y' = -\frac{1}{2}$.
Equation of the tangent line:
at $(0, -2)$: $y = -2(x - 0) - 2$ $\quad$ or $\quad$ $y = -2x - 2$
at $(3, 1)$: $y = -\frac{1}{2}(x - 3) + 1$ $\quad$ or $\quad$ $y = -\frac{1}{2}x + \frac{5}{2}$

5. (10 pts; compare p113 #19, 21) Find these limits (using the algebraic method).

$$\lim_{x \to 3} \frac{x^2 - 2x - 3}{x^2 - 4x + 3} = \lim_{x \to 3} \frac{(x - 3)(x + 1)}{(x - 3)(x - 1)} = \lim_{x \to 3} \frac{x + 1}{x - 1} = \frac{4}{2} = 2$$

$$\lim_{x \to -2} \frac{x^2 - 9}{x^2 + 9} = \frac{(-2)^2 - 9}{(-2)^2 + 9} = \frac{-5}{13}$$

6. (7 pts; compare p155 #3) Given the cost function $C(x) = 0.01x^2 + 1.2x + 60$ and revenue function $R(x) = 5x$, find the profit function $P(x)$. Find the marginal profit when $x = 100$.

$P(x) = R(x) - C(x) = 5x - (.01x^2 + 1.2x + 60) = 3.8x - .01x^2 - 60$
$P'(x) = 3.8 - .02x$
$P'(100) = 3.8 - .02(100) = 3.8 - 2 = 1.8$

7. (8 pts; compare p155 #1) Given the distance function $s(t) = t + t^4$, find the velocity $v(t)$ and the acceleration $a(t)$. Find the velocity and acceleration when $t = 2$.

$v(t) = s'(t) = 1 + 4t^3 \qquad v(2) = 1 + 4(2)^3 = 33$

$a(t) = v'(t) = s''(t) = 12t^2 \qquad a(2) = 12(2)^2 = 48$

8. (a) (5 pts)Complete the definition: $f'(x) = \lim_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$

(b) (10 pts; compare p137 #12) **Use the limit definition** to find $f'(x)$, for $f(x) = x^2 - 2x$.

$$\begin{aligned} f'(x) &= \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{[(x+h)^2 - 2(x+h)] - [x^2-2x]}{h} \\ &= \lim_{h\to 0} \frac{x^2+2xh+h^2-2x-2h-x^2+2x}{h} = \lim_{h\to 0} \frac{2xh+h^2-2h}{h} \\ &= \lim_{h\to 0} \frac{(h)(2x+h-2)}{h} = \lim_{h\to 0} 2x+h-2 = 2x-2 \end{aligned}$$

(c) (10 pts; p138 #31) **Use the limit definition** to find $f'(x)$, for $f(x) = \dfrac{x}{1+x}$.

Check your answer using by using the quotient rule.

Using the quotient rule: $f'(x) = \dfrac{(1)(1+x) - (x)(1)}{(1+x)^2} = \dfrac{1+x-x}{(1+x)^2} = \dfrac{1}{(1+x)^2}$

Using the limit definition:

$$\begin{aligned} f'(x) &= \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{\dfrac{(x+h)}{1+(x+h)} - \dfrac{x}{1+x}}{h} = \lim_{h\to 0} \frac{1}{h}\left[\frac{(x+h)}{1+(x+h)} - \frac{x}{1+x}\right] \\ &= \lim_{h\to 0} \frac{1}{h}\left[\frac{(x+h)(1+x)}{(1+(x+h))(1+x)} - \frac{x(1+x+h)}{(1+x)(1+x+h)}\right] = \lim_{h\to 0} \frac{1}{h}\left[\frac{x+h+x^2+hx-x-x^2-xh}{(1+(x+h))(1+x)}\right] \\ &= \lim_{h\to 0} \frac{1}{h}\left[\frac{h}{(1+(x+h))(1+x)}\right] = \lim_{h\to 0} \frac{1}{(1+(x+h))(1+x)} = \frac{1}{(1+x)^2} \end{aligned}$$

Grading scale: 90–100 A (10); 80–89 B (13); 65–79 C (29); 55–64 D (14); below 55 F (21).

The class average was about 70.

You should note that these problems were either assigned homework problems or very similar to ones that were assigned. If you can do most of the homework (and really understand it), you should be able to get a C or better in the course.