MATH 211, Section C | Review for EXAM 2 | scheduled for **Friday, 3/21/2003**
Prof. J. Beachy | Sections 2.9, 3.1–3.5 | **Bring a valid picture ID**

Sections 3.1–3.3

The main idea is to use the sign of the first derivative and the sign of the second derivative of a function to provide useful information about the graph of the function.

$$\begin{array}{ccccc} f'(x) > 0 & \longleftrightarrow & \text{positive slope} & \longleftrightarrow & f(x) \text{ is increasing} \\ f'(x) < 0 & \longleftrightarrow & \text{negative slope} & \longleftrightarrow & f(x) \text{ is decreasing} \end{array}$$

$$\begin{array}{ccccc} f''(x) > 0 & \longleftrightarrow & f'(x) \text{ is increasing} & \longleftrightarrow & f(x) \text{ is concave up} \\ f''(x) < 0 & \longleftrightarrow & f'(x) \text{ is decreasing} & \longleftrightarrow & f(x) \text{ is concave down} \end{array}$$

These correspondences allow us to find relative maximum and relative minimum values of a function, and provide help in graphing because they say something about the shape of the graph of a function.

In Sections 3.1–3.3 you should review the definitions and theorems (look for the boxes of text that are shaded in grey). These include the definitions of *increasing* and *decreasing* (p 187), *critical point* (p 188), *relative maximum* and relative minimum (p 189), *concave up* and *concave down* (p 202), *point of inflection* (p 208), *vertical asymptote* (p 221), and *horizontal asymptote* (p 222). Review Theorem 2 about relative extreme values (p 190), the First Derivative Test for Relative Extrema (p 192), the Second Derivative Test for Relative Extrema (p 204), and the Strategy for Sketching Graphs (p 226).

A strategy for sketching graphs *(You can follow this outline instead of the author's, if you choose.)*

(a) Classify the function: Polynomial, Rational, or Other

(b) Find vertical and horizontal asymptotes (if the function is rational).

(c) Find $f'(x)$ and $f''(x)$, simplify, and factor (if possible).

(d) Use $f'(x)$ to find the critical points (if any), and where the graph is increasing or decreasing.

(e) Use $f''(x)$ to find the points of inflection (if any), and where the graph is concave up or down.

(f) Find the relative extreme points (if any). If the second derivative is relatively nice, use the second derivative test to check whether the points you found are relative maxima or relative minima. If the second derivative looks complicated, go back to the first derivative and use the first derivative test.

(g) Plot the critical points and points of inflection (if they exist), and then plot a few nearby points. (The goal is to use calculus to find the important parts of the graph, and then plot more points in these regions.) If there are asymptotes, sketch the lines. Finally, sketch the graph.

Sections 3.4–3.5

In many applications, the variable x is defined only on an interval, rather than for all real numbers. Section 3.4 shows how to find the absolute maximum and minimum values of a function on a closed interval. This is used in Section 3.5 in problems of various types. (Note: you can omit pp 257–260, on minimizing inventory costs.)

A few formulas to remember: profit equals revenue minus cost; the area of a rectangle is length times width; the volume of a box is length times width times height; the area of a circle is pi times the square of the radius ($A = \pi r^2$); the circumference of a circle is pi times its diameter ($c = \pi d$).

A strategy for solving Max–Min problems *(Compare the author's suggestions on p250.)*

1. *Identify*: Write down in words what it is you are asked to minimize or maximize.

2. *Model*: Find a function that describes the quantity in step 1. To do that you need to write the quantity as a function of x, where x represents something you can treat as an independent variable. The problem may tell you what to use as x, or it may give you a big clue if you read it carefully.

3. *(Experiment)*: If you have trouble writing down an equation for the function you need, try one or two simple cases by actually putting in some numbers. This may help you see what varies and what stays fixed, so that you can make the right choice for x and see how it can be used to compute values of the function.

4. *Simplify*: Do any algebraic simplification you can. If at first you seemed to need to use two variables to express the quantity in step 1, then you must use information in the problem to eliminate all but one variable. (Our calculus techniques from earlier sections require a function of *one* variable.)

5. *Solve*: Use the calculus techniques we have developed to find the maximum and minimum values. *Note*: Be sure to use either the first or second derivative test to make certain that you actually found the maximum or minimum value that was asked for.