MATH 211 FINAL EXAM 5/3/97 NO CALCULATORS!

1. (30 points) Find the derivative of each of these functions. You do not need to simplify your answers.

(a) $f(x) = x^2 + \dfrac{1}{x^2}$ (b) $f(x) = (x^5 + \ln x)^4$ (c) $f(x) = \dfrac{x^3 - 1}{x^2 + 1}$ (d) $f(x) = e^{x^2}\sqrt{x^3 + 5}$

2. (15 pts) Find the following limits. (a) $\lim\limits_{x \to 6} \dfrac{x^2 - 4x - 12}{x - 6} =$ (b) $\lim\limits_{x \to 3} \dfrac{x^2 - 4x - 12}{x^2 - 8x + 12} =$

3. (20 pts) Let $f(x) = \dfrac{1}{2}x + \dfrac{2}{x}$.

(a) Find and classify the extreme points (relative maximum and minimum points) of $y = f(x)$.
(b) Graph the given function $f(x)$, using your knowledge of calculus.

4. (30 pts) Find the following integrals.

(a) $\displaystyle\int_1^4 \frac{1}{x^2}\, dx =$ (b) $\displaystyle\int_0^5 e^{-2t}\, dt =$ (c) $\displaystyle\int \frac{x^2 - 2x}{x^3 - 3x^2 + 1}\, dx =$

5. (15 pts) Find the area bounded by the curves $y = 2x^2 + x - 5$ and $y = x + 3$ and the vertical lines $x = -2$ and $x = 2$. (A sketch of the curves may help you.)

6. (15 pts) A store manager wants to build a 600-square-foot rectangular enclosure. Three sides of the enclosure will be built of redwood fencing, at a cost of \$14 per foot. The fourth side will be built of cement blocks, at a cost of \$28 per foot. Find the length and width of the enclosure that will minimize the total cost of the building materials.

7. (15 pts) The demand equation for a certain product is $p = 200 - 3x$, where p is the price and x is the number of units produced. The cost function is $C(x) = 75 + 80x - x^2$, where $0 \le x \le 40$.

(a) Determine the level of production that will maximize the profit.
(b) Suppose that the government imposes a tax of \$4 per unit produced. Find the new price that maximizes the profit.

8. (15 pts) Five grams of a certain radioactive material decays to 3 grams in 1 year. After how many years will just 1 gram remain?

9. (7 pts) Let $P(t) = P_0 e^{kt}$. Compute $P'(t)$ and show that $P'(t) = kP(t)$.

10. (8 pts) Find the equation of the tangent line to the curve $y = x \ln x$ at $x = 1$.

11. (20 pts) Find the derivative of each of the following functions.

(a) $f(x) = \sqrt{x + \sqrt{x}}$ (b) $f(x) = \ln\left((x^2 + 3)^5 (x^3 + 1)^{-4}\right)$

12. (10 pts) Using the limit definition of the derivative (not the power formula), find $f'(x)$ if $f(x) = x^3$.