

1. (7 points; Similar to assigned homework p.30 #28) Find the composite function $f(g(x))$, where $f(x) = \frac{1}{1-x} + \frac{1}{x}$ and $g(x) = x + 1$. *Simplify your answer.*

$$\begin{aligned} f(g(x)) &= \frac{1}{1-g(x)} + \frac{1}{g(x)} = \frac{1}{1-(x+1)} + \frac{1}{x+1} = \frac{1}{-x} + \frac{1}{x+1} = \frac{1(x+1)}{-x(x+1)} + \frac{(-x)1}{(-x)(x+1)} \\ &= \frac{x-1-x}{-x(x+1)} = \frac{-1}{-x(x+1)} \end{aligned}$$

2. (8 points; Example 2, page 49) A rectangular box has a square copper base, wooden sides, and a wooden top. The copper costs \$21 per square foot and the wood costs \$2 per square foot.

(a) Write an expression giving the surface area of the box, in terms of its dimensions.

The base is square, so if x is the width and h is the height, then the surface area is $2x^2 + 4xh$.

Comment: It is important to use the fact that the base is square, so that only two variables are required instead of three. I took off 3 points for not making that simplification.

(b) Write an expression giving the total cost of the materials used to make the box, in terms of its dimensions.

The total cost is $\$21x^2 + \$2x^2 + \$2(4xh)$, for the base, top, and 4 sides. The total is $23x^2 + 8xh$ dollars.

3. (10 points) On the axes below, graph the two functions $y = -\frac{1}{4}x + 1$ and $y = \frac{1}{x}$.

Comments: See Example 9 on page 9 for the graph of $y = 1/x$. This is a basic function, and anyone who passes a college algebra course should be able to graph it. I was disappointed in the performance of the class, especially since this was one of the important functions I noted on the review sheet.

The line should be tangent to the curve at the point $(2, 1/2)$. In fact, I chose this function because it is the solution to problem 5 (a). The problem is similar to assigned homework problem #37 on p.70.

4. (15 points) Find the derivative of each of the following functions. Use the power rule—do *not* use the limit definition of the derivative.

(a) (3 pts) If $f(x) = x^{10}$, then $f'(x) = 10x^9$.

(b) (4 pts; like p.85 #14) If $f(x) = \frac{1}{\sqrt[5]{x}} = x^{-1/5}$, then $f'(x) = -1/5x^{-6/5}$.

(c) (4 pts; p85 #39) $\frac{d}{dx} (x^{3/4}) = 3/4x^{-1/4}$.

(d) (4 pts; see p85 #18 and #25) Find the slope of the curve $y = x^3$ at $x = \frac{1}{4}$.

The slope at x is given by the derivative $\frac{dy}{dx} = 3x^2$, so the answer is found by substituting $x = 1/4$, to get $3/16$.

5. (10 pts) (a) (5 pts; Example 2 p.81) Find the equation of the line tangent to the curve $y = \frac{1}{x}$ at $x = 2$.

See page 81 of the text for the solution.

(b) (5 pts; see p.77 #27 and #28) Find the point on the graph of $y = x^2$ where the tangent line is parallel to the line $2x - y = 4$.

Put the line in slope-intercept form to find its slope. This gives $y = 2x - 4$, so we need to find the point on the graph for which the slope is equal to 2. The slope of the curve is given by the derivative $\frac{dy}{dx} = 2x$, so we need to solve $2x = 2$. The solution is $x = 1$, and the corresponding point on the curve $y = x^2$ is $(1, 1)$.

6. (10 points; p.57 #25) A florist shop estimates that its weekly revenue is $R(x) = 21x$ dollars, where x is the number of sales per week. If the corresponding cost function is $C(x) = 9x + 800$ dollars, find the profit function $P(x)$ (Profit is the difference between revenue and cost.) How much profit is made when sales are at 100 per week?

$$P(x) = R(x) - C(x) = 21x - (9x + 800) = 12x - 800.$$

$$P(100) = 12(100) - 800 = 400.$$

7. (15 pts) Find the following limits.

(6 pts; like p96 #23) Substituting $x = 4$ gives the form $\frac{0}{0}$, so you must simplify by factoring $x - 4$ out of the numerator and denominator.

$$\lim_{x \rightarrow 4} \frac{x^2 - 5x + 4}{x^2 - 3x - 4} = \lim_{x \rightarrow 4} \frac{(x-4)(x-1)}{(x-4)(x+1)} = \lim_{x \rightarrow 4} \frac{x-1}{x+1} = \frac{3}{5}$$

Comment: When you cancel $x - 4$ from the numerator and denominator, you get a different functions, which is not equal to the original one. However, the limits of the two functions are the same, so as long as you continue to use the limit symbol, you do have equality. I took off one point for not using the limit symbols in an appropriate way. Mathematical notation is a language, and you must use the correct grammar as you learn the new vocabulary of limits and derivatives.

(4 pts; like p.96 #14) First substitute $x = -3$. The numerator and denominator are nonzero, so no simplification is necessary.

$$\lim_{x \rightarrow -3} \frac{\sqrt{x+3} - 3}{\sqrt{x+3} + 3} = \frac{\sqrt{-3+3} - 3}{\sqrt{-3+3} + 3} = \frac{-3}{3} = -1$$

(5 pts; like p97 #56) Divide numerator and denominator by the highest power of x in the denominator. Then you can see what happens to the limit, because $\lim_{x \rightarrow +\infty} \frac{1}{x} = 0$ and $\lim_{x \rightarrow +\infty} \frac{1}{x^2} = 0$.

$$\lim_{x \rightarrow +\infty} \frac{2x^2 - x + 3}{3x^2 + x + 2} = \lim_{x \rightarrow +\infty} \frac{2 - 1/x + 3/x^2}{3 + 1/x + 2/x^2} = \frac{2}{3}$$

8. (25 points)

(a) (5 pts) Complete the limit definition of the derivative of a function $f(x)$:

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

(b) (10 pts; p97 #39, for $x = 1$ instead of $x = 0$) Use the limit definition of the derivative of a function to find $f'(1)$, for the function $f(x) = (x+1)^3$.

$$\begin{aligned} f'(1) &= \lim_{h \rightarrow 0} \frac{f(1+h) - f(1)}{h} = \lim_{h \rightarrow 0} \frac{(1+h+1)^3 - (1+1)^3}{h} = \lim_{h \rightarrow 0} \frac{(2+h)^3 - (2)^3}{h} \\ &= \lim_{h \rightarrow 0} \frac{8 + 12h + 6h^2 + h^3 - 8}{h} = \lim_{h \rightarrow 0} \frac{(12 + 6h + h^2)(h)}{h} = \lim_{h \rightarrow 0} 12 + 6h + h^2 = 12 \end{aligned}$$

(c) Use the limit definition of the derivative of a function to find $f'(x)$, for the function $f(x) = \frac{1}{x^2}$.

Comment: the solution should be in your class notes from 2/3/99. Also look at page 84.

$$\begin{aligned} f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \rightarrow 0} \frac{1}{h} \left(\frac{1}{(x+h)^2} - \frac{1}{x^2} \right) = \lim_{h \rightarrow 0} \frac{1}{h} \left(\frac{(1)(x^2)}{(x+h)^2(x^2)} - \frac{(x+h)^2(1)}{(x+h)^2(x^2)} \right) \\ &= \lim_{h \rightarrow 0} \frac{1}{h} \left(\frac{x^2 - (x^2 + 2xh + h^2)}{(x+h)^2(x^2)} \right) = \lim_{h \rightarrow 0} \frac{1}{h} \left(\frac{-2xh - h^2}{(x+h)^2(x^2)} \right) = \lim_{h \rightarrow 0} \frac{-2x - h}{(x+h)^2(x^2)} = \frac{-2x}{(x)^2(x^2)} = \frac{-2}{x^3} \end{aligned}$$