MATH 211, Section C — Review for EXAM 3 — 11/11/2002

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Exam 3, covering Sections 3.4–3.5 and 4.1–4.4, will be given on **Friday, November 15**. Bring a valid picture I.D.

Sections 3.4–3.5

In many applications, the variable x is defined only on an interval, rather than for all real numbers. Section 3.4 shows how to find the absolute maximum and minimum values of a function on a closed interval. This is used in Section 3.5 in problems of various types. (Note: you can omit pp 257–260, on minimizing inventory costs.)

A few formulas to remember: profit equals revenue minus cost; the area of a rectangle is length times width; the volume of a box is length times width times height; the area of a circle is pi times the square of the radius; the perimeter of a circle is pi times its diameter.

A Strategy for solving Max–Min problems (see p 250 for the author's suggestions)

1. *Identify*: Write down in words what it is you are asked to minimize or maximize.
2. *Model*: Find a function that describes the quantity in step 1. To do that you need to write the quantity as a function of x, where x represents something you can treat as an independent variable. The problem may tell you what to use as x, or it may give you a big clue if you read it carefully.
3. *(Experiment)*: If you have trouble writing down an equation for the function you need, try one or two simple cases by actually putting in some numbers. This may help you see what varies and what stays fixed, so that you can make the right choice for x and see how it can be used to compute values of the function.
4. *Simplify*: Do any algebraic simplification you can. If at first you seemed to need to use two variables to express the quantity in step 1, then you must use information in the problem to eliminate all but one variable. (Our calculus techniques from earlier sections require a function of *one* variable.)
5. *Solve*: Use the calculus techniques we have developed to find the maximum and minimum values. *Note*: Be sure to use either the first or second derivative test to make certain that you actually found the maximum or minimum value that was asked for.

Sections 4.1–4.4

The derivative of $f(x) = a^x$ is $f'(x) = ka^x$, where $k = f'(0)$. If we choose the base e, then $f(x) = e^x$ has derivative $f'(x) = e^x$, because $f'(0) = 1$. That is the reason for using $e = \lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n = 2.7182818.....$ The connection between the exponential function e^x and the natural logarithm function $\ln x$ is given by the two equations $e^{\ln x} = x$ and $\ln e^x = x$, which express the fact that the functions are inverses of each other. This means that their graphs are symmetric about the line $y = x$ (see the important graph at the bottom of page 307). Facts to know:

$$\ln 1 = 0 \qquad \ln e = 1 \qquad e^{\ln x} = x \qquad \ln\left(e^{u(x)}\right) = u(x) \qquad e^{u(x)}e^{v(x)} = e^{u(x)+v(x)} \qquad \left(e^{u(x)}\right)^k = e^{ku(x)}$$

$$\ln u(x)^k = k \ln u(x) \qquad \ln\left(u(x)v(x)\right) = \ln u(x) + \ln v(x) \qquad \ln\left(\frac{u(x)}{v(x)}\right) = \ln u(x) - \ln v(x) \qquad \text{(see p 305)}$$

Essential differentiation formulas: (also review the product rule and quotient rule)

Chain rule: $\frac{d}{dx}\, f(u(x)) = f'(u(x))\, u'(x)$ Extended power rule: $\frac{d}{dx}\,(u(x))^k = k(u(x))^{k-1}u'(x)$

Exponentials: $\frac{d}{dx}\, e^x = e^x$ $\frac{d}{dx}\, e^{u(x)} = e^{u(x)}\, u'(x)$ Natural logs: $\frac{d}{dx}\,\ln x = \frac{1}{x}$ $\frac{d}{dx}\,\ln u(x) = \frac{1}{u(x)}\, u'(x)$

Applications: The model for "uninhibited growth" (see p 318) is based on the assumption that the rate of growth is proportional to the amount, at any time. That is, $\frac{dP}{dt} = kP(t)$, at time t. The solution is

$$P(t) = P_0 e^{kt}\ ,$$

where P_0 is the initial amount (when $t = 0$). The problems usually have enough information to find P_0 and k, and then you can answer additional questions about the situation. The same equation is used for radioactive decay (in this case k is negative). Sometimes the half-life is given, and this can be used to find the decay constant k.

Review problems

3.4 #25,35,65,67; 3.5 #19,27,29,33; 4.1 #21,29,59,61; 4.2 #23–28,51,55,57,79,89,91; 4.3 #17,33; 4.4 #7,25,31,35