MATH 229 Work Sheet
OPTIMIZATION PROBLEMS

1. Show that the rectangle that has maximum area for a given perimeter is a square.
2. Find the area of the largest rectangle with lower base on the $x$-axis and upper vertices on the curve $y = 12 - x^2$.
3. A poster is to contain 50 square inches of printed matter, with margins of 4 in. each at top and bottom and 2 in. at each side. Find the over-all dimensions if the total area of the poster is as small as possible.
4. An open box is to be made from a 2-foot by 3-foot rectangular piece of material by cutting equal squares from the corners and turning up the sides. Find the volume of the largest box that can be made this way.
5. A net enclosure for a golf driving range is open at one end, which is square. The volume of the enclosure is $83\frac{1}{3}$ cubic meters. Find the dimensions that require the least amount of netting. (To visualize the enclosure, think of a box with square ends but no top, cut out one of the ends, and turn the box upside down.)
6. An indoor phisycal fitness room consists of a rectangular region with a semicircle at each end. The perimeter of the room is to be a 200-meter jogging track, while the rectangular region will contain the fitness machines. Find the dimensions that will make the area of the rectangular region as large as possible.
7. Cans are to be manufactured to contain a fixed volume $V$, with the usual shape (a right circular cylinder). Assume that there is no waste involved cutting the vertical sides of the can, but that the circular top and bottom are cut from a square and the part trimmed off is wasted, so it must be included in the total cost. Find the ratio of height to diameter for the most economical cans.
8. Show that the volume of the largest right circular cylinder that can be inscribed in a given right circular cone is $\frac{4}{9}$ the volume of the cone.

Answers:

1. Let $P$ be the perimeter, and let $x$ be the length of the base. The area is $A(x) = \dfrac{P}{2}x - x^2$, and this function has a maximum at $x = \dfrac{P}{4}$.
2. 32
3. 9 in. wide by 18 in. high
4. Let $x$ be the width of the square you cut out. Then the volume of the box is $V(x) = 4x^3 - 10x^2 + 6x$, and the volume is maximized by choosing $x = (5 - \sqrt{7})/6 \approx .392$ (to 3 decimal places). The corresponding volume of the box is $V \approx 1.056$.
5. Let $x$ be length of the square end. The surface area is $A(x) = x^2 + \dfrac{250}{x}$, and it is minimized by choosing $x = 5$. The remaining dimension is $10/3$.
6. Let $x$ be the length of the rectangle. Then the area of the rectangular region is $A(x) = \frac{200}{\pi}x - \frac{2}{\pi}x^2$, and it is maximized by setting $x = 50$. The corresponding width is $100/\pi \approx 31.831$.
7. The ratio of height to diameter should be $4/\pi \approx 1.273$.