Exam 2 is scheduled for Friday, October 27, 2006. It will cover Sections 3.5–3.10 and 4.1–4.3.

You need to know these formulas: $\frac{d}{dx}\,[f(x)g(x)] = f'(x)g(x) + f(x)g'(x)$ $\quad \frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{f'(x)g(x) - f(x)g'(x)}{g(x)^2}$

Section 3.5: $\frac{d}{dx}\,(\sin x) = \cos x$ $\quad \frac{d}{dx}\,(\cos x) = -\sin x$ $\quad \frac{d}{dx}\,(\tan x) = \sec^2 x$ $\quad \frac{d}{dx}\,(\sec x) = \sec x \tan x$

Many of the limits reduce to $\lim\limits_{x\to 0} \frac{\sin x}{x} = 1$

Section 3.6: Chain rule: $\frac{d}{dx}\, f(u(x)) = f'(u(x))u'(x)$ $\quad \frac{d}{dx}\,(u(x))^n = n(u(x))^{n-1}u'(x)$

If $y = f(u(x))$, the chain rule can also be written as $\frac{dy}{dx} = \frac{dy}{du}\,\frac{du}{dx}$

Section 3.7: Given an equation in the variables x and y, instead of solving for y in terms of x and then finding y', you can think of y as an *implicit* function of x and use the chain rule to differentiate both sides of the equation with respect to x. This gives a way to solve for y', but the answer is then typically given in terms of both x and y.

Section 3.8: Depending on the function, you can repeat the process of taking a derivative. Notation: $f''(x)$, $f'''(x)$, etc, or $\frac{d^2y}{dx^2} = \frac{d}{dx}\left[\frac{dy}{dx}\right]$, $\frac{d^3y}{dx^3} = \frac{d}{dx}\left[\frac{d^2y}{dx^2}\right]$, etc. If $s = f(t)$ gives the position of an object as a function of time, then the first derivative $f'(t)$ gives its velocity, and the second derivative $f''(t)$ gives its acceleration. We can write $v = f'(t)$, and $a = f''(t)$, or $v = \frac{ds}{dt}$, and $a = \frac{dv}{dt} = \frac{d^2s}{dt^2}$.

Section 3.9: Here is the typical related rates problem. If y is a function of x, and x is a function of the time t, then the chain rule says that $\frac{dy}{dt} = \frac{dy}{dx}\,\frac{dx}{dt}$. Remember that $\frac{dy}{dt}$ is the rate of change of y with respect to time, and $\frac{dx}{dt}$ is the rate of change of x with respect to time. If you know one of these two rates, and can calculate $\frac{dy}{dx}$, then you can find the other rate. In some of the problems, when you find an equation relating x and y, it may be easier to use implicit differentiation (with respect to time) to find an equation relating $\frac{dy}{dt}$ and $\frac{dx}{dt}$, rather than first solving for y in terms of x, and then differentiating.

Section 3.10: Using the way we have constructed tangent lines, if x is close to a, then the corresponding points on the graph of $y = f(x)$ are close to the tangent line. In this section, the author writes the tangent line at $(a, f(a))$ in the form $L(x) = f(a) + f'(a)(x - a)$, and calls it the *linearization* of $f(x)$ at a. When x is close to a, written $x \approx a$, then $f(x)$ is approximated by the linearization at a, so $f(x) \approx f(a) + f'(a)(x - a)$. If you are only interested in approximating the change in $f(x)$, then all you need to use is the slope of the tangent line: if $\Delta x = x - a$ is the change in x, and $\Delta y = f(x) - f(a)$ is the corresponding change in y, then $\Delta y \approx f'(a)\Delta x$. The author uses differential notation, so that $\Delta y \approx dy$, where $dy = f'(x)dx$ is the *differential* of y, in which dx plays the role of Δx.

Section 4.1: A *critical number* c of a function $f(x)$ is a value of x for which either $f'(c) = 0$ or $f'(c)$ is undefined. To search for local maximum and local minimum values of the function, we only need to look at the critical numbers of the function. To find absolute maximum and absolute minimum values of a function on an interval $[a, b]$, we must look at $f(c)$ for all critical numbers c *and* at $f(a)$ and $f(b)$.

Section 4.2: The Mean Value Theorem (p 235) is used primarily to prove other results. It says (roughly) that if a function $f(x)$ has a derivative, then in any interval $[a, b]$ there is some number c where the instantaneous rate of change $f'(c)$ is the same as the average rate of change $\frac{f(b) - f(a)}{b - a}$. One application says that if two functions have the same derivative, then their graphs are "parallel" (differ by a constant amount).

Section 4.3: If $f'(x) > 0$, then $f(x)$ is *increasing* (as x increases); if $f'(x) < 0$, then $f(x)$ is *decreasing*. (Model: linear functions.) Knowing the sign of $f'(x)$ helps to graph $y = f(x)$, and helps to decide when a critical number produces a local maximum and when it produces a local minimum (see the First Derivative Test on p 241).

If $f''(x) > 0$, then the graph of $y = f(x)$ is *concave up*, since its slopes are increasing; if $f''(x) < 0$, then the graph of $y = f(x)$ is *concave down*, since its slopes are decreasing. (Model: quadratic functions.) Knowing the sign of $f''(x)$ helps to graph $y = f(x)$, and if c is a critical number with $f''(c) > 0$, then $f(c)$ is a local minimum since $f(x)$ is concave up; if $f''(c) < 0$, then $f(c)$ is a local maximum since $f(x)$ is concave down (See the Second Derivative Test on p 245.)