1. (20 points) Find the derivative $f'(x)$ or $\dfrac{dy}{dx}$.

(a) (p156 #35) $f(x) = x\sqrt{x} - \dfrac{1}{x^2\sqrt{x}} = x^{3/2} - x^{-5/2}$ $\quad f'(x) = \dfrac{3}{2}x^{1/2} + \dfrac{5}{2}x^{-7/2}$

(b) (p183 #19) $\dfrac{d}{dx}(2x-5)^4(8x^2-5)^{-3} = 4(2x-5)^3(2)(8x^2-5)^{-3} + (2x-5)^4(-3)(8x^2-5)^{-2}(16x)$

(c) (p183 #42) $y = \sqrt{\cos(\sin^2 x)} = (\cos((\sin x)^2))^{1/2}$ $\quad y' = \frac{1}{2}(\cos((\sin x)^2))^{-1/2}(-\sin((\sin x)^2)(2\sin x\cos x)$

(d) (p190 #51) $x^2y^2 + xy = 2$ *Simplify your answer.* Differentiate both sides with respect to x.

$(2x)y^2 + x^2(2yy') + y + xy' = 0$ $\qquad (2x^2y + x)y' = -2xy^2 - y$

$$y' = \frac{-2xy^2 - y}{2x^2y + x} = \frac{(-y)(2xy+1)}{(x)(2xy+1)} = -\frac{y}{x}$$

2. (5 points; p197 #17) Find the second derivative $f''(x)$ for $f(x) = \dfrac{1}{\sqrt{1-x}} = (1-x)^{-1/2}$.

$f'(x) = (-\frac{1}{2})(1-x)^{-3/2}(-1) = \frac{1}{2}(1-x)^{-3/2}$ $\qquad f''(x) = (\frac{1}{2})(-\frac{3}{2})(1-x)^{-5/2}(-1) = \frac{3}{4}(1-x)^{-5/2}$

3. (6 points; p172) Write out the steps that show that $\displaystyle\lim_{x\to 0} \frac{\cos x - 1}{x} = 0$. See the text.

4. (6 points; p239 #14) For the function $f(x) = \dfrac{x}{x+2}$, find all solutions of the equation $f'(x) = \dfrac{f(b)-f(a)}{b-a}$ in the interval $[a, b]$, where $a = 1$, $b = 4$.

$f'(x) = \dfrac{(x+2)-x}{(x+2)^2} = \dfrac{2}{(x+2)^2}$ $\qquad \dfrac{f(b)-f(a)}{b-a} = \dfrac{\frac{2}{3}-\frac{1}{3}}{4-1} = \dfrac{1}{3} \div 3 = \dfrac{1}{3}\cdot\dfrac{1}{3} = \dfrac{1}{9}$

We need to solve the equation $\dfrac{2}{(x+2)^2} = \dfrac{1}{9}$. $\qquad 18 = (x+2)^2$ $\qquad x + 2 = \pm 3\sqrt{2}$

5. (6 points; p211 #25) For $y = \cos x$, find the differential dy and evaluate dy for $x = \pi/6$ and $dx = 0.05$.

$dy = -\sin x\, dx$ $\qquad dy = -\sin(\pi/6)(.05) = -.5(.05) = -.025$

6. (7 points; p211 #34) Find the linearization $L(x)$ of $f(x) = x^6$ at $a = 2$ and use it to approximate $(1.97)^6$.

$L(x) = f(2) + f'(2)(x-2) = 64 + 192(x-2)$ $\qquad$ since $2^6 = (2^3)(2^3) = 64$ and $6(2^5) = 6(32) = 192$.

$(1.97)^6 \approx 64 + 192(1.97-2) = 64 + (192)(-.03) = 64 - 5.76 = 58.24$

7. (7 points; p187 Ex2) Find the equation of the tangent line to the curve $x^3 + y^3 = 6xy$ at the point $(3, 3)$.

See page 187 of the text.

8. (18 pts) For $f(x) = -x^3 - 3x^2 + 4$, graph the function after finding
(a) the intervals on which f is increasing or decreasing;

$f'(x) = -3x^2 - 6x = (-3x)(x + 2)$ $\qquad$ $f'(x) = 0$ for $x = -2$ and for $x = 0$

By looking at the factors, $f'(x)$ is negative for $x < -2$, positive for $-2 < x < 0$, and negative for $0 < x$. Conclusion: $f(x)$ is decreasing on $(-\infty, -2)$ and $(0, \infty)$ and increasing on $(-2, 0)$.

(b) the local maximum and minimum values of f; $\qquad$ Local min at $(-2, 0)$ and local max at $(0, 4)$

(c) the intervals of concavity and the inflection points. $\qquad$ $f''(x) = -6x - 6 = -6(x + 1)$

The graph is concave up on $(-\infty, -1)$ and concave down on $(-1, \infty)$, so $(-1, 2)$ is a point of inflection.

Now graph the function, including the points $(-3, 4)$, $(-2, 0)$, $(-1, 2)$, $(0, 4)$, $(1, 0)$.

9. (13 points; p248 #32) For the function $f(x) = (x^2 - 1)^3$, find
(a) the intervals on which f is increasing or decreasing;

$f'(x) = 3(x^2 - 1)^2(2x) = 6x(x^2 - 1)^2$ $\qquad$ $(x^2 - 1)^2 \geq 0$ so $f'(x)$ is negative for $x < 0$, positive for $x > 0$
Conclusion: $f(x)$ is decreasing on $(-\infty, 0)$ and increasing on $(0, \infty)$.

(b) the local maximum and minimum values of f;

Setting $f'(x) = 0$, we have $(6x)(x^2 - 1)^2 = 0$, so either $x = 0$ or $x^2 = 1$, and the solutions are $x = -1, 0, 1$.
Using the answer from part (a), we can see that although $(-1, 0)$, $(0, -1)$, and $(1, 0)$ are critical points, the point $(0, -1)$ is a local minimum, and the other two points are neither local max nor local min.

(c) the intervals of concavity and the inflection points.

$f''(x) = 6(x^2 - 1)^2 + 6x(2)(x^2 - 1)(2x) = 6(x^2 - 1)[(x^2 - 1) + 4x^2] = 6(x^2 - 1)(5x^2 - 1)$
Setting $f''(x) = 0$ we get $x^2 - 1 = 0$ or $5x^2 - 1 = 0$, so $x = \pm 1$ or $x = \pm 1/\sqrt{5}$.
Looking at signs of the factors of $f''(x)$, or testing $f''(x)$ at points in these intervals, you can show that $f''(x)$ is positive for $-\infty < x < -1$, for $-1/\sqrt{5} < x < 1/\sqrt{5}$, and for $1 < x < \infty$. You can show that $f''(x)$ is negative for $-1 < x < -1/\sqrt{5}$ and for $1/\sqrt{5} < x < 1$.
Conclusion: $f(x)$ is concave up on the intervals $(-\infty, -1)$, $(-1/\sqrt{5}, 1/\sqrt{5})$, and $(1, \infty)$, and concave down on the intervals $(-1, -1/\sqrt{5})$ and $(1/\sqrt{5}, 1)$.

10. (12 points; p199 Ex 1) Air is being pumped into a spherical balloon so that its volume increases at a rate of 100 cm^3/sec. How fast is the radius of the balloon increasing when the diameter is 50 cm? *Hint:* The volume of a sphere is $V = \frac{4}{3}\pi r^3$.

See page 199 of the text.