

1. (5 pts; p 23 #21) For the function  $f(x) = x - x^2$ , find (and simplify)  $\frac{f(x+h) - f(x)}{h}$ .

$$\begin{aligned} \frac{f(x+h) - f(x)}{h} &= \frac{[(x+h) - (x+h)^2] - [x - x^2]}{h} = \frac{x+h - x^2 - 2xh - h^2 - x + x^2}{h} \\ &= \frac{h - 2xh - h^2}{h} = \frac{h(1 - 2x - h)}{h} = 1 - 2x - h \end{aligned}$$

2. (5 pts; p 80 #23)  $\lim_{x \rightarrow 5^+} \frac{6}{x-5} = +\infty$

Explanation: Substituting  $x = 5$  gives  $\frac{6}{0}$ , which shows that there should be a vertical asymptote. Since  $x$  approaches 5 from the right, the values of  $x$  that we are interested in are greater than 5, so the denominator is always positive, and therefore the fraction is always positive. This shows that the limit from the right is positive infinity.

3. (5 pts; p 90 #18)  $\lim_{x \rightarrow 1} \frac{x^3 - 1}{x^2 - 1} = \lim_{x \rightarrow 1} \frac{(x-1)(x^2 + x + 1)}{(x-1)(x+1)} = \lim_{x \rightarrow 1} \frac{(x^2 + x + 1)}{(x+1)} = \frac{3}{2}$

4. (5 pts; p 91 #46) Let  $f(x) = \begin{cases} 4 - x^2 & \text{if } x \leq 2 \\ x - 1 & \text{if } x > 2 \end{cases}$  Find  $\lim_{x \rightarrow 2^-} f(x)$  and  $\lim_{x \rightarrow 2^+} f(x)$ .

$$\lim_{x \rightarrow 2^-} f(x) = \lim_{x \rightarrow 2} 4 - x^2 = 4 - 2^2 = 0$$

$$\lim_{x \rightarrow 2^+} f(x) = \lim_{x \rightarrow 2} x - 1 = 2 - 1 = 1$$

Conclusion: (you weren't asked to answer this)  $\lim_{x \rightarrow 2} f(x)$  does not exist