

1. (10 pts; p 132 #7) Use the *limit definition* of the derivative to find $f'(2)$ for the function $f(x) = 3x^2 - 5x$. Then use $f'(2)$ to find the equation of the line tangent to the parabola $y = 3x^2 - 5x$ at the point $(2, 2)$.

Solution: The derivative of $f(x) = 3x^2 - 5x$ at $x = 2$ will give the slope of the tangent line.

$$\begin{aligned} f'(2) &= \lim_{h \rightarrow 0} \frac{f(2+h) - f(2)}{h} = \lim_{h \rightarrow 0} \frac{[3(2+h)^2 - 5(2+h)] - [3(2)^2 - 5(2)]}{h} && 4/10 \text{ points for this step} \\ &= \lim_{h \rightarrow 0} \frac{3(4 + 4h + h^2) - 10 - 5h - 12 + 10}{h} = \lim_{h \rightarrow 0} \frac{12h + 3h^2 - 5h}{h} = \lim_{h \rightarrow 0} \frac{7h + 3h^2}{h} \\ &= \lim_{h \rightarrow 0} \frac{h(7 + 3h)}{h} = \lim_{h \rightarrow 0} 7 + 3h = 7 \end{aligned}$$

To find the tangent line, use the point-slope form of the equation of a line: $y = m(x - a) + b$. We were given $a = 2$, $b = 2$, and now we have found $m = 7$. The equation of the tangent line is $y = 7(x - 2) + 2$.

2. (10 pts; p 144 #25) Use the *limit definition* of the derivative to find the derivative $g'(x)$ of the function $g(x) = \sqrt{1 + 2x}$.

$$\begin{aligned} f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \rightarrow 0} \frac{\sqrt{1 + 2(x+h)} - \sqrt{1 + 2x}}{h} && 4/10 \text{ points for this step} \\ &= \lim_{h \rightarrow 0} \frac{(\sqrt{1 + 2x + 2h} - \sqrt{1 + 2x})(\sqrt{1 + 2x + 2h} + \sqrt{1 + 2x})}{h(\sqrt{1 + 2x + 2h} + \sqrt{1 + 2x})} \\ &= \lim_{h \rightarrow 0} \frac{(1 + 2x + 2h) - (1 + 2x)}{h(\sqrt{1 + 2x + 2h} + \sqrt{1 + 2x})} = \lim_{h \rightarrow 0} \frac{(1 + 2x + 2h - 1 - 2x)}{h(\sqrt{1 + 2x + 2h} + \sqrt{1 + 2x})} \\ &= \lim_{h \rightarrow 0} \frac{2h}{h(\sqrt{1 + 2x + 2h} + \sqrt{1 + 2x})} = \lim_{h \rightarrow 0} \frac{2}{\sqrt{1 + 2x + 2h} + \sqrt{1 + 2x}} \\ &= \frac{2}{\sqrt{1 + 2x} + \sqrt{1 + 2x}} = \frac{2}{2\sqrt{1 + 2x}} = \frac{1}{\sqrt{1 + 2x}} \end{aligned}$$