Math 229, Review for Exam I, September 29, 2006 Prof. J. Beachy

The test will cover Sections 1.1–1.3, 2.1–2.6, 3.1–3.4. **No calculators will be allowed.** This review sheet gives a summary of the most important definitions and theorems. You must know the statements of Definition 3.2.2 (the definition of the derivative on page 134) and Definition 2.4.2 (the definition of a limit on page 93). You must know how to use the other definitions and theorems.

The Limit of a Function

Definition 2.4.2 (page 93): $\lim_{x\to a} f(x) = L$ if for every $\epsilon > 0$ there is a $\delta > 0$ such that $|f(x) - L| < \epsilon$ whenever $0 < |x - a| < \delta$.

One-sided limits (pages 75, 96): A similar definition can be given when x approaches a from just one side. Then $\lim_{x\to a} f(x) = L$ if and only if both one-sided limits exist and $\lim_{x\to a^-} f(x) = L$ and $\lim_{x\to a^+} f(x) = L$.

Infinite limits (pages 77, 99): $\lim_{x\to a^-} f(x) = \pm\infty$ and $\lim_{x\to a^+} f(x) = \pm\infty$ are best thought of in terms of a vertical asymptote at $x = a$ (for the graph of $y = f(x)$).

Limit laws (pages 82–88): Assume that $\lim_{x\to a} f(x)$ and $\lim_{x\to a} g(x)$ are defined and finite. Then

$$\lim_{x\to a} f(x) \pm g(x) = \lim_{x\to a} f(x) \pm \lim_{x\to a} g(x) \qquad \lim_{x\to a} cf(x) = c\lim_{x\to a} f(x)$$

$$\lim_{x\to a} f(x)\cdot g(x) = \lim_{x\to a} f(x)\cdot \lim_{x\to a} g(x) \qquad \lim_{x\to a} \frac{f(x)}{g(x)} = \frac{\lim_{x\to a} f(x)}{\lim_{x\to a} g(x)}, \text{ provided } \lim_{x\to a} g(x) \neq 0$$

$$\lim_{x\to a} [f(x)]^n = \left[\lim_{x\to a} f(x)\right]^n \qquad \lim_{x\to a} \sqrt[n]{f(x)} = \sqrt[n]{\lim_{x\to a} f(x)} \quad \text{where } n \text{ is a positive integer.}$$

If $f(x)$ is a polynomial, then $\lim_{x\to a} f(x) = f(a)$, for all real numbers a.

If $f(x)$ is a rational function, then $\lim_{x\to a} f(x) = f(a)$, for all a in the domain of $f(x)$.

If $f(x) \le g(x) \le h(x)$ and $\lim_{x\to a} f(x) = \lim_{x\to a} h(x)$, then $\lim_{x\to a} g(x) = \lim_{x\to a} f(x) = \lim_{x\to a} h(x)$.

Continuous Functions

Definition 2.5.1 (page 102): A function $f(x)$ is **continuous** at $x = a$ if (i) $f(a)$ is defined; (ii) $\lim_{x\to a^-} f(x)$ exists; (iii) $\lim_{x\to a^+} f(x)$ exists; (iv) $\lim_{x\to a^-} f(x) = \lim_{x\to a^+} f(x) = f(a)$

Theorem 2.5.4 (page 105): If $f(x)$ and $g(x)$ are continuous at $x = a$, then so are $f(x) \pm g(x)$, $cf(x)$, and $f(x)g(x)$. If $g(a) \neq 0$, then the quotient function $f(x)/g(x)$ is continuous at $x = a$.

Theorem 2.5.5 (page 105): Any polynomial function is continuous for all real numbers; any rational function is continous for all real numbers in its domain.

Continuous Functions (continued)

Theorem 2.5.8 (page 107): If $f(x)$ is continuous, and $\lim_{x\to a} g(x)$ exists, then

$$\lim_{x\to a} f(g(x)) = f\left(\lim_{x\to a} g(x)\right).$$

Theorem 2.5.10 [Intermediate Value Theorem] (page 109): If $f(x)$ is continuous on the closed interval $[a, b]$, and N is any number between $f(a)$ and $f(b)$, then the equation $f(x) = N$ has a solution between a and b.

The Definition of the Derivative of a Function

Definition 3.2.2 (page 134): The **derivative** of a function $f(x)$ is defined by

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}, \quad \text{if this limit exists.}$$

Interpretation of the derivative as the slope (page 128): The derivative $f'(a)$ gives the slope of the line tangent to the curve $y = f(x)$ at the point $(a, f(a))$. The equation of the tangent line at $(a, f(a))$ is given by

$$y = f'(a)(x - a) + f(a).$$

Interpretation of the derivative as a rate of growth (page 130): The derivative $f'(a)$ gives the instantaneous rate of growth of the function $y = f(x)$ (as compared to x), when $x = a$. If $y = f(x)$ is the position of an object at time x, then $f'(a)$ gives the velocity of the object at time $x = a$.

Theorem 3.2.4 (page 140): If $f(x)$ is *not* continuous at $x = a$, then $f'(x)$ is *not* defined at $x = a$.

Differentiation Formulas

These are the differentiation formulas you should know (pages 146–153). Assume that $f(x)$ and $g(x)$ are differentiable functions, and remember that $f'(x)$ is also written as $\frac{df}{dx}$.

$$\frac{d}{dx}(mx + b) = m \qquad \frac{d}{dx}(x^n) = nx^{n-1} \quad \text{(for any real number } n\text{)}$$

$$\frac{d}{dx}[cf(x)] = cf'(x) \qquad \frac{d}{dx}[f(x) + g(x)] = f'(x) + g'(x)$$

$$\frac{d}{dx}[f(x)g(x)] = f'(x)g(x) + f(x)g'(x) \qquad \frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{f'(x)g(x) - f(x)g'(x)}{g(x)^2}$$