**MATH 240** **TEST I** 9/22/97

Prof. John Beachy  Show all of the work necessary to justify your answers.

1. (20 pts) (a) When solving a system of linear equations, explain why adding a multiple of one equation to another does not change the solution set.

(b) Use Gauss-Jordan reduction to solve the following linear system.

$$\begin{array}{rrrrcr} x_1 & +2x_2 & & +2x_4 & = & 1 \\ -x_1 & -2x_2 & +x_3 & -x_4 & = & -4 \\ x_1 & +2x_2 & +x_3 & +4x_4 & = & -3 \\ 2x_1 & +4x_2 & +2x_3 & +7x_4 & = & -5 \end{array}$$

2. (10 pts) Find the inverse of the matrix $A = \begin{bmatrix} 1 & a & b \\ 0 & 1 & c \\ 0 & 0 & 1 \end{bmatrix}$, where $a, b, c$ are any real numbers.

3. (10 pts) Express the matrix $\begin{bmatrix} 2 & 7 \\ 1 & 3 \end{bmatrix}$ as a product of elementary matrices.

4. (10 pts) If $A$ is a nonsingular $n \times n$ matrix, show that $A^T$ is also nonsingular, and find a formula for the inverse of $A^T$ in terms of the inverse of $A$.

5. (10 pts) Let $A, B$ be nonsingular $n \times n$ matrices.

(a) Given $AB = BA$, prove that $(AB)^{-1} = A^{-1}B^{-1}$.

(b) Given $(AB)^{-1} = A^{-1}B^{-1}$, prove that $AB = BA$.

6. (30 pts) Determine whether the given subset $W$ is a subspace of the vector space $V$. (In each part, either check that all three of the necessary conditions hold, or give a numerical counterexample to one of them.)

(a) Let $V = \mathbf{R}^3$ and let $W = \{(x, y, z) \mid y = 2x - z\}$.

(b) Let $V = \mathbf{R}^2$ and let $W = \{(x, y) \mid x^2 + y^2 \leq 1\}$.

(c) Let $V$ be the vector space $P_3$ of all polynomials of degree at most 3. Let $W$ be the set of all polynomials $p(x)$ in $P_3$ for which $\int_0^1 p(x)\,dx = 0$.

7. (10 pts) In $\mathbf{R}^2$, use ordinary addition $(x_1, y_1) + (x_2, y_2) = (x_1 + x_2,\, y_1 + y_2)$, but define a new scalar multiplication by $r \cdot (x, y) = (rx, -ry)$. Check each of the four laws $M_1$, $M_2$, $M_3$, $M_4$ for scalar multiplication. If the law is valid, give a proof. If not, give a numerical counterexample to show that it fails.