MATH 240 HOMEWORK 4 NAME ______________________

Prof. J. Beachy Friday, 9/20/02 Score / 20

1. (10 pts) In $\mathbf{R}^2$, define a new addition $(x_1, y_1) \oplus (x_2, y_2) = (x_1 + x_2 - 1,\ y_1 + y_2)$, and define a new scalar multiplication by $r * (x, y) = (rx - r + 1, ry)$.

(a) Prove that the distributive law $r * (\mathbf{v}_1 \oplus \mathbf{v}_2) = r * \mathbf{v}_1 \oplus r * \mathbf{v}_2$ holds for all scalars r and all vectors $\mathbf{v}_1 = (x_1, y_1)$ and $\mathbf{v}_2 = (x_2, y_2)$ in $\mathbf{R}^2$.

(b) Prove the the associative law $r * (s * \mathbf{v}_1) = (rs) * \mathbf{v}_1$ holds for all scalars r, s and all vectors $\mathbf{v}_1 = (x_1, y_1)$ in $\mathbf{R}^2$.

2. (10 pts) Let V be the vector space M_n of all $n \times n$ matrices. Let W be the subset of V consisting of all matrices X for which $X^T = -X$, where X^T is the transpose of X. Check that W is a subspace of V.