

The score on this set of problems will replace the lowest quiz or homework score.

1. (7 pts) Let A be the following matrix. $A = \begin{bmatrix} 1 & -1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 2 & 3 \\ 2 & -2 & -1 & 0 & -1 \\ -3 & 3 & 0 & -3 & -3 \end{bmatrix}$

(a) Reduce the matrix A to row echelon form.

(b) Find a basis for the column space of A .

(c) Find a basis for the nullspace of A .

2. (7 pts) Let \mathcal{P}_2 be the vector space of all polynomials of degree at most 2. Define the function $T : \mathcal{P}_2 \rightarrow \mathcal{P}_2$ by $T(p(x)) = p(x) + x^2 p''(x)$, for all polynomials $p(x)$ in \mathcal{P}_2 .

(a) Show that T is a linear transformation.

(b) Find the matrix of T relative to the standard basis $\mathcal{B} = \{x^2, x, 1\}$.

(c) Find the rank and nullity of the matrix you found in part (b).

3. (6 pts) For each of the following subsets, decide whether or not the subset is a subspace of the given vector space. If it *is* a subspace, show that it satisfies the necessary conditions. If it is *not* a subspace, explain why not.

(a) $\{(x, y, z) \mid 2x - 3y + 4z = 0\}$ in \mathbf{R}^3 .

(b) $\{p(x) \mid p(0) = 2\}$ in the vector space \mathcal{P}_2 of all polynomials of degree at most 2.

(c) The set of all diagonal 2×2 matrices in the vector space \mathcal{M}_2 of all 2×2 matrices.