MATH 240 QUIZ 2 NAME ______________________

Prof. J. Beachy Friday, 9/6/2002

1. (8 points; p39 #25) For the matrix $A = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$, find the inverse $A^{-1}$. Check that $AA^{-1} = I$ and $A^{-1}A = I$.

The determinant of $A$ is $(2)(2) - (1)(1) = 3$, so $A^{-1} = \frac{1}{3}\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} \frac{2}{3} & -\frac{1}{3} \\ -\frac{1}{3} & \frac{2}{3} \end{bmatrix}$.

Check: $\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} \frac{2}{3} & -\frac{1}{3} \\ -\frac{1}{3} & \frac{2}{3} \end{bmatrix} = \begin{bmatrix} \frac{4}{3} - \frac{1}{3} & -\frac{2}{3} + \frac{2}{3} \\ +\frac{2}{3} - \frac{2}{3} & -\frac{1}{3} + \frac{4}{3} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Similarly $\begin{bmatrix} \frac{2}{3} & -\frac{1}{3} \\ -\frac{1}{3} & \frac{2}{3} \end{bmatrix}\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

2. (6 points) Given the matrix $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$, find all $2 \times 2$ matrices $X$ such that $AX = XA$.

For $X = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, we have $\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} c & d \\ 0 & 0 \end{bmatrix}$ and $\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & a \\ 0 & c \end{bmatrix}$, and these are equal only if $c = 0$ and $a = d$.

On the other hand, any matrix with this form satisfies the equation since
$\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} a & b \\ 0 & a \end{bmatrix} = \begin{bmatrix} 0 & a \\ 0 & 0 \end{bmatrix}$ and $\begin{bmatrix} a & b \\ 0 & a \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & a \\ 0 & 0 \end{bmatrix}$.

3. (6 points; p38 #15) If $A$ is any matrix, explain why $AA^T$ must be a symmetric matrix.

To prove that $AA^T$ is symmetric, we will show that taking its transpose leaves it unchanged. (Remember that $(XY)^T = Y^T X^T$.)

$(AA^T)^T = (A^T)^T A^T = AA^T$