MATH 240 QUIZ 8 NAME ____________________

Prof. J. Beachy Friday, 11/8/02 Score / 20

1. (10 pts) Let $L : \mathcal{P}_2 \to \mathcal{P}_1$ be defined by $L(a_2t^2 + a_1t + a_0) = (a_2 + a_0)t + (a_1 - a_0)$. Consider the ordered bases $S = \{t^2, t, 1\}$ and $S' = \{t^2 + t, t + 1, 1\}$ for $\mathcal{P}_2$ and $T = \{t, 1\}$ and $T' = \{t + 1, t + 3\}$ for $\mathcal{P}_1$.

(a) Find the matrix representation $M_{T \leftarrow S}(L)$ of L with respect to S and T.

(b) Find the transition matrices $P_{S \leftarrow S'}$ from S' to S and $P_{T' \leftarrow T}$ from T to T'.

(c) Use parts (a) and (b) to find the matrix representation $M_{T' \leftarrow S'}(L)$ of L with respect to S' and T'.

This is Example 1 on page 300 of the text, disguised as a linear transformation $L : \mathcal{P}_2 \to \mathcal{P}_1$ instead of $L : \mathbf{R}^3 \to \mathbf{R}^2$. The matrices will be exactly the same.

2. (5 pts) Find all values of t that make this determinant equal to zero. $\begin{vmatrix} t-1 & 0 & 1 \\ -2 & t & -1 \\ 0 & 0 & t+1 \end{vmatrix}$

The best solution is probably via the algorithm for 3×3 matrices. $\left|\begin{matrix} t-1 & 0 & 1 \\ -2 & t & -1 \\ 0 & 0 & t+1 \end{matrix}\right|\begin{matrix} t-1 & 0 \\ -2 & t \\ 0 & 0 \end{matrix}$

$$\begin{vmatrix} t-1 & 0 & 1 \\ -2 & t & -1 \\ 0 & 0 & t+1 \end{vmatrix} = (t-1)(t)(t+1) + (0)(-1)(0) + (1)(-2)(0) - (0)(t)(1) - (0)(-1)(t-1) - (t+1)(-2)(0)$$

$= (t-1)(t)(t+1)$. Setting the determinant equal to zero gives $t = -1, 0, 1$.

3. (5 pts) Find the value of this determinant. $\begin{vmatrix} 4 & 2 & 2 & 0 \\ 2 & 0 & 0 & 0 \\ 3 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{vmatrix} = 2 \begin{vmatrix} 2 & 1 & 1 & 0 \\ 2 & 0 & 0 & 0 \\ 3 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{vmatrix} = 4 \begin{vmatrix} 2 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 3 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{vmatrix}$

$$= -4 \begin{vmatrix} 1 & 0 & 0 & 0 \\ 2 & 1 & 1 & 0 \\ 3 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{vmatrix} = -4 \begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{vmatrix} = 4 \begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix} = 4$$