MATH 240 QUIZ 9 NAME ____________________

Prof. J. Beachy Friday, 11/15/02 Score / 20

1. (6 pts) Find the area of the parallelogram whose vertices in $\mathbf{R}^2$ are $(1,1)$, $(2,4)$, $(6,2)$, $(7,5)$.

The parallelogram is determined by the vectors $(2,4)-(1,1)=(1,3)$ and $(6,2)-(1,1)=(5,1)$. The area is the absolute value of the determinant $\begin{vmatrix} 1 & 5 \\ 3 & 1 \end{vmatrix} = -14$.

You can also use the author's method: $\begin{vmatrix} 1 & 1 & 1 \\ 2 & 4 & 1 \\ 6 & 2 & 1 \end{vmatrix} = (4+6+4)-(24+2+2) = -14$.

2. (8 pts) Find the adjoint of the matrix $\begin{bmatrix} a & b & 0 \\ c & d & 0 \\ x & y & 1 \end{bmatrix}$.

Answer: $\begin{bmatrix} d & -b & 0 \\ -c & a & 0 \\ cy-dx & -(ay-bx) & ad-bc \end{bmatrix}$

3. (6 pts) Show that if A is any $n \times n$ matrix, then $\det(\text{adj}(A)) = (\det(A))^{n-1}$.

The basic identity is $A \cdot \text{adj}(A) = \det(A) \cdot I$, where I is the $n \times n$ identity matrix. From this we get

$\det(A \cdot \text{adj}(A)) = \det(\det(A) \cdot I)$

$\det(A) \cdot \det(\text{adj}(A)) = (\det(A))^n$, since $\det(\det(A) \cdot I)$ is the product of terms on the main diagonal.

If $\det(A) \neq 0$, we can divide both sides of the equation by $\det(A)$ to get $\det(\text{adj}(A)) = (\det(A))^{n-1}$.

If $\det(A) = 0$, then we can see from the basic identity that $\text{adj}(A)$ must be singular, and therefore $\det(\text{adj}(A)) = 0$. Then the equation $\det(\text{adj}(A)) = (\det(A))^{n-1}$ certainly holds.