MATH 240 **TEST I** 9/23/2002

Prof. John Beachy Show all of the work necessary to justify your answers.

1. (15 pts) (a) Use Gauss-Jordan reduction to solve the following linear system.

$$\begin{array}{rrrcr} x_1 & +2x_2 & +3x_3 & = & 6 \\ 2x_1 & -3x_2 & +2x_3 & = & 14 \\ 3x_1 & +x_2 & -x_3 & = & -2 \end{array}$$

(b) Explain why the answer to part (a) shows how to write the vector $\mathbf{b} = \begin{bmatrix} 6 \\ 14 \\ -2 \end{bmatrix}$ as a linear combination of the vectors $\mathbf{v_1} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$, $\mathbf{v_2} = \begin{bmatrix} 2 \\ -3 \\ 1 \end{bmatrix}$, and $\mathbf{v_3} = \begin{bmatrix} 3 \\ 2 \\ -1 \end{bmatrix}$.

2. (10 pts) Find the inverse of the matrix $A = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 2 \end{bmatrix}$.

3. (15 pts) Write the matrix $A = \begin{bmatrix} 1 & -2 \\ 2 & -1 \end{bmatrix}$ as a product of elementary matrices. Then write A^{-1} as a product of elementary matrices. (*Hint:* You can do this without finding A^{-1}.)

4. (15 pts) Show that if A is a nonsingular matrix and $AB = AC$ for matrices B, C, then $B = C$. Give an example to show that this can fail for 2×2 matrices if A is singular.

5. (30 pts) Determine whether the given subset W is a subspace of the vector space V. (In each part, either check that all three of the necessary conditions hold, or give a numerical counterexample to one of them.)

(a) Let $V = \mathbf{R}^3$ and let $W = \{(x, y, z) \mid z = x + 2y\}$.

(b) Let $V = \mathbf{R}^2$ and let $W = \{(x, y) \mid y \geq 0\}$. (the first and second quadrants)

(c) Let V be the vector space M_{44} of all 4×4 matrices, and let W be the set of all symmetric 4×4 matrices.

6. (15 pts) In $\mathbf{R}^2$, use ordinary scalar multiplication $r \cdot (x, y) = (rx, ry)$ but define a new addition of vectors by $(x_1, y_1) \oplus (x_2, y_2) = (x_1 + x_2, 2y_1 + y_2)$. Check each of these laws: (1) commutative law for addition, (2) associative law for addition, and (5), (6) distributive laws. If the law is valid, give a proof. If not, give a numerical counterexample to show that it fails.