1. Let $A$ be the following matrix. $A = \begin{bmatrix} 1 & 2 & 3 & 6 \\ 2 & -3 & 2 & 14 \\ 1 & -5 & -1 & 8 \\ 3 & 1 & -1 & -2 \end{bmatrix}$

   (a) Find the reduced row echelon form of $A$.

   (b) Find the rank and nullity of $A$.

   (c) Find a basis for the row space of $A$.

   (d) Find a basis for the column space of $A$.

   (e) Find a basis for the nullspace of $A$.

2. Let $M_{22}$ be the vector space of all $2 \times 2$ matrices, and let $Q = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$.

   (a) Let $W$ be the set of all matrices $A$ in $M_{22}$ such that $AQ = 0$. Show that $W$ is a subspace of $M_{22}$.

   (b) Find a basis for $W$, and find the dimension of $W$.

3. Let $S = \{(1, 0, 0), (1, 1, 0), (1, 1, 1)\}$ and $T = \{(1, -1, 0), (0, 1, -1), (0, 0, 1)\}$ be ordered bases for $\mathbb{R}^3$. Let $v = (3, 2, 1)$.

   (a) Find the coordinate vector $[v]_T$ of $v$ with respect to the basis $T$.

   (b) Find the transition matrix $P_{S \rightarrow T}$.

   (c) Use $P_{S \rightarrow T}$ to find the coordinate vector $[v]_S$ of $v$ with respect to the basis $S$.

4. Let $\{v_1, v_2, v_3\}$ be linearly independent vectors in $\mathbb{R}^n$. Prove that if $A$ is a nonsingular $n \times n$ matrix, then the vectors $\{Av_1, Av_2, Av_3\}$ are linearly independent in $\mathbb{R}^n$.

5. Find $\det(A)$ by row-reducing $A$ to an upper triangular matrix, for the matrix $A = \begin{bmatrix} 2 & 0 & -1 & 7 \\ 6 & 1 & 0 & 4 \\ 8 & -2 & 1 & 0 \\ 4 & 1 & 0 & 2 \end{bmatrix}$.

6. Show that $\begin{vmatrix} x - 4 & 0 & -1 \\ 2 & x - 1 & 0 \\ 2 & 0 & x - 1 \end{vmatrix} = (x - 1)(x - 2)(x - 3)$.

7. Suppose that $A$ and $B$ are similar matrices. Explain why $\det(A) = \det(B)$.

8. Find the adjoint of $A = \begin{bmatrix} a & b & c \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$. Check that $A \cdot \text{adj}(A) = \det(A) \cdot I_3$. 