1. Let \( L : \mathbb{R}^5 \to \mathbb{R}^4 \) be the linear transformation defined by \( L(x) = Ax \), for the matrix
   \[
   A = \begin{bmatrix}
   1 & 0 & -1 & 3 & -1 \\
   1 & 0 & 0 & 2 & -1 \\
   2 & 0 & -1 & 5 & -1 \\
   0 & 0 & -1 & 1 & 0 \\
   \end{bmatrix}
   \]. Given that \( A \) row-reduces to
   \[
   \begin{bmatrix}
   1 & 0 & 0 & 2 & 0 \\
   0 & 0 & 1 & -1 & 0 \\
   0 & 0 & 0 & 0 & 1 \\
   0 & 0 & 0 & 0 & 0 \\
   \end{bmatrix},
   \]
   (a) find a basis for \( \ker L \);
   (b) find a basis for \( \text{range } L \);
   (c) find \( \dim(\ker L) \) and \( \dim(\text{range } L) \).

2. Define the linear transformation \( L : P_2 \to P_2 \) by \( L(p(t)) = p(t) + 2p'(t) \).
   (a) Find the matrix \( M_{S \to S}(L) \) of \( L \) relative to the standard basis \( S = \{t^2, t, 1\} \).
   (b) Find the matrix \( M_{T \to T}(L) \) of \( L \) relative to the basis \( T = \{t^2 + t + 1, t + 1, 1\} \).

3. Let \( L : \mathbb{R}_3 \to \mathbb{R}_3 \) be the linear transformation defined by \( L(x_1, x_2, x_3) = (x_1, x_2 + 2x_3, 2x_2 + x_3) \). Let \( S = \{(1,0,0), (0,1,0), (0,0,1)\} \) be the standard basis for \( \mathbb{R}_3 \), and let \( T \) be the basis \( \{(1,0,0), (0,\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}), (0,-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})\} \).
   (a) Find the matrix \( M_{S \to S}(L) \) of \( L \) with respect to the basis \( S \).
   (b) Find the matrix \( M_{T \to T}(L) \) of \( L \) with respect to the basis \( T \).
   (c) Find the transition matrices \( P_{S \to T} \) and \( P_{T \to S} \) that change coordinates.
   (d) Check that \( M_{T \to T}(L) = P_{T \to S} \cdot M_{S \to S}(L) \cdot P_{S \to T} \).

4. Show that if the \( n \times n \) matrix \( B \) is similar to the matrix \( A \), then \( B^T \) is similar to \( A^T \).

5. Let \( W \) be the subspace of \( \mathbb{R}_4 \) spanned by the vectors \((1, -1, 1, 1)\) and \((1, 0, 2, 1)\). Use the Gram-Schmidt process to find an orthonormal basis for \( W \).

6. Let \( M_{22} \) be the vector space of all \( 2 \times 2 \) matrices. For \( A, B \) in \( M_{22} \), define an inner product by \( (A, B) = \text{tr}(B^T A) \).
   (Recall: \( \text{tr}(A) \) denotes the trace of \( A \), which is the sum of entries on the main diagonal.)
   (a) Check that \( (A, B) = (B, A) \) for all \( A, B \) in \( M_{22} \).
   (b) For any \( 2 \times 2 \) matrix \( A \), check that \( (A, A) = 0 \) if and only if \( A = 0 \).

7. Answer EITHER part A OR part B.
   A. If \( u \) and \( v \) are vectors in an inner product space \( V \), prove the parallelogram law, which states that
      \[ ||u + v||^2 + ||u - v||^2 = 2||u||^2 + 2||v||^2 \].
   B. Let \( S = \{u_1, u_2, \ldots, u_n\} \) be an orthogonal set of nonzero vectors in an inner product space \( V \). Show that \( S \)
      is a linearly independent set.

8. Find the orthogonal complement in \( \mathbb{R}^4 \) of the subspace \( W \) spanned by the vectors \((1, 1, 0, 0), (0, 1, 1, 0), (0, 0, 1, 1)\).