MATH 240 QUIZ 2 NAME Solutions

Prof. J. Beachy Friday, 9/7/2007 Score / 20

1. (5 points; 1.3 p32 #33) Write this linear system in matrix form.

$$\begin{array}{rcrcl} 2x_1 & + & 3x_2 & = & 0 \\ 3x_2 & + & x_3 & = & 0 \\ 2x_1 & - & x_2 & = & 0 \end{array}$$

$$\begin{bmatrix} 2 & 3 & 0 \\ 0 & 3 & 1 \\ 2 & -1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

2. (5 points; part of 1.4 p41 #34) Let $A = \begin{bmatrix} c & 0 \\ 0 & c \end{bmatrix}$. Show that $AB = BA$ for any 2×2 matrix $B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$.

$AB = \begin{bmatrix} c & 0 \\ 0 & c \end{bmatrix} \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} = \begin{bmatrix} cb_{11} & cb_{12} \\ cb_{21} & cb_{22} \end{bmatrix}$ and $BA = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} \begin{bmatrix} c & 0 \\ 0 & c \end{bmatrix} = \begin{bmatrix} b_{11}c & b_{12}c \\ b_{21}c & b_{22}c \end{bmatrix}$

$AB = BA$ since $cb_{11} = b_{11}c$, $cb_{12} = b_{12}c$, $cb_{21} = b_{21}c$, and $cb_{22} = b_{22}c$.

3. (5 points; 1.5 p52 #24 (b)) Assume that A and B are symmetric matrices.

(a) If $AB = BA$, explain why AB is a symmetric matrix.

Since A and B are symmetric, we have $A^T = A$ and $B^T = B$.

$(AB)^T = B^T A^T = BA$ and then $(AB)^T = AB$ since $BA = AB$. This shows that AB is symmetric.

(b) If AB is a symmetric matrix, explain why it must be true that $AB = BA$.

If AB is symmetric, then $(AB)^T = AB$. This means that $BA = B^T A^T = (AB)^T = AB$.

4. (5 points; 1.5 p53 #33 (b)) For the matrix $A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$, find the inverse A^{-1}. Check that $AA^{-1} = I_2$ and $A^{-1}A = I_2$.

You can find the inverse by solving the matrix equation $\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} w & x \\ y & z \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$.

It's easier if you remember this formula: $\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \dfrac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$, provided $ad - bc \neq 0$.

The determinant of A is $(1)(1) - (2)(2) = -3$, so $A^{-1} = -\dfrac{1}{3} \begin{bmatrix} 1 & -2 \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} -\frac{1}{3} & \frac{2}{3} \\ \frac{2}{3} & -\frac{1}{3} \end{bmatrix}$.

Check: $\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} -\frac{1}{3} & \frac{2}{3} \\ \frac{2}{3} & -\frac{1}{3} \end{bmatrix} = \begin{bmatrix} -\frac{1}{3} + \frac{4}{3} & -\frac{2}{3} + \frac{2}{3} \\ +\frac{2}{3} - \frac{2}{3} & \frac{4}{3} - \frac{1}{3} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Similarly $\begin{bmatrix} -\frac{1}{3} & \frac{2}{3} \\ \frac{2}{3} & -\frac{1}{3} \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$