MATH 240 QUIZ 6 NAME Solutions

Prof. J. Beachy Friday, 10/12/2007 Score / 20

1. (8 pts) Find a basis for the null space of the matrix $\begin{bmatrix} 1 & 2 & 3 & -1 \\ 2 & 3 & 2 & 0 \\ 3 & 4 & 1 & 1 \\ 1 & 1 & -1 & 1 \end{bmatrix}$.

The first step is to row-reduce the matrix.

$$\begin{bmatrix} 1 & 2 & 3 & -1 \\ 2 & 3 & 2 & 0 \\ 3 & 4 & 1 & 1 \\ 1 & 1 & -1 & 1 \end{bmatrix} \rightsquigarrow \begin{bmatrix} 1 & 2 & 3 & -1 \\ 0 & -1 & -4 & 2 \\ 0 & -2 & -8 & 4 \\ 0 & -1 & -4 & 2 \end{bmatrix} \rightsquigarrow \begin{bmatrix} 1 & 0 & -5 & 3 \\ 0 & 1 & 4 & -2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

This gives us the solution $x_1 = 5x_3 - 3x_4$, $x_2 = -4x_3 + 2x_4$. First set $x_3 = 1, x_4 = 0$, and then set $x_3 = 0, x_4 = 1$. The following vectors are a basis for the null space.

$$\begin{bmatrix} 5 \\ -4 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -3 \\ 2 \\ 0 \\ 1 \end{bmatrix}$$

2. (12 pts) Let $S = \{[1\ \ 2], [0\ \ 1]\}$ and $T = \{[1\ \ 1], [2\ \ 3]\}$.
Find the transition matrix $P_{S \leftarrow T}$ that changes from coordinates relative to T to coordinates relative to S.
For $\mathbf{v} = [1\ \ 3]$, you are given that $[\mathbf{v}]_T = \begin{bmatrix} -3 \\ 2 \end{bmatrix}$. Find $[\mathbf{v}]_S$ by computing $P_{S \leftarrow T} \cdot [\mathbf{v}]_T$, and then check your answer.

To find $P_{S \leftarrow T}$, put the vectors in as columns in the appropriate matrix and reduce: $[S \mid T] \rightsquigarrow [I \mid P_{S \leftarrow T}]$.

$$\left[\begin{array}{cc|cc} 1 & 0 & 1 & 2 \\ 2 & 1 & 1 & 3 \end{array}\right] \rightsquigarrow \left[\begin{array}{cc|cc} 1 & 0 & 1 & 2 \\ 0 & 1 & -1 & -1 \end{array}\right] \qquad P_{S \leftarrow T} = \begin{bmatrix} 1 & 2 \\ -1 & -1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 \\ -1 & -1 \end{bmatrix} \begin{bmatrix} -3 \\ 2 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

To check your answer you only need to check that $[1\ \ 3] = 1 \cdot [1\ \ 2] + 1 \cdot [0\ \ 1]$, and this is indeed true.