MATH 240 QUIZ 9 NAME Solutions

Prof. J. Beachy Friday, 11/9/2007 Score / 20

1. (10 pts) Let W be the subspace of R_4 spanned by the vectors $(1,-1,1,1)$ and $(1,0,2,1)$. Use the Gram-Schmidt process to find an orthonormal basis for W.

Let $\mathbf{u}_1 = (1,-1,1,1)$ and $\mathbf{u}_2 = (1,0,2,1)$.

Then $\mathbf{v_1} = (1,-1,1,1)$, and $\mathbf{v}_2$ is $\mathbf{u}_2$ minus its projection onto $\mathbf{v}_1$.

$$\mathbf{v}_2 = \mathbf{u}_2 - \frac{\mathbf{u}_2 \cdot \mathbf{v}_1}{\mathbf{v}_1 \cdot \mathbf{v}_1}\mathbf{v}_1 = (1,0,2,1) - \frac{(1+0+2+1)}{(1+1+1+1)}(1,-1,1,1) = (1,0,2,1) - (1,-1,1,1) = (0,1,1,0).$$

Finally, divide each vector by its length: $\mathbf{w}_1 = \frac{1}{2}(1,-1,1,1)$ and $\mathbf{w}_2 = \frac{1}{\sqrt{2}}(0,1,1,0)$.

2. (10 pts; §5.5 #14) Let W be the plane in $\mathbf{R}^3$ given by the equation $x + y - 2z = 0$, and let $\mathbf{v} = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$. Find vectors $\mathbf{w}$ in W and $\mathbf{u}$ in $W^\perp$ with $\mathbf{v} = \mathbf{w} + \mathbf{u}$. Then find the distance from $\mathbf{v}$ to W.

The normal vector $\mathbf{n} = \begin{bmatrix} 1 \\ 1 \\ -2 \end{bmatrix}$ determines $W^\perp$, so it is easiest to find $\mathbf{u}$ instead of $\mathbf{w}$, by finding its projection onto $\mathbf{n}$. Since $\mathbf{n}$ does not have length 1, use the formula $\mathbf{u} = \frac{\mathbf{v} \cdot \mathbf{n}}{\mathbf{n} \cdot \mathbf{n}}\mathbf{n} = \frac{3}{6}\mathbf{n} = \frac{1}{2}\begin{bmatrix} 1 \\ 1 \\ -2 \end{bmatrix} = \begin{bmatrix} 1/2 \\ 1/2 \\ -1 \end{bmatrix}$.

The distance from $\mathbf{v}$ to W is the length of the perpendicular vector that goes from $\mathbf{v}$ to W. That is, the distance is the length of the projection onto $W^\perp$. The distance from $\mathbf{v}$ to W is $||\mathbf{u}|| = \sqrt{(\frac{1}{2})^2 + (\frac{1}{2})^2 + (-1)^2} = \sqrt{\frac{6}{4}} = \frac{\sqrt{6}}{2}$.

Note: To double check your answer, note that $\mathbf{w} = \mathbf{v} - \mathbf{u} = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} - \begin{bmatrix} 1/2 \\ 1/2 \\ -1 \end{bmatrix} = \begin{bmatrix} 1/2 \\ -1/2 \\ 0 \end{bmatrix}$, *and then you can easily check that* $\mathbf{w}$ *belongs to the given plane. You can see that it is easiest to find* $\mathbf{u}$ *first, rather than finding an orthonormal basis for* W *so that you can find the projection of* $\mathbf{v}$ *onto* W.