MATH 240 Properties of the Real Numbers

This is a list of some of the properties of the set of real numbers that we need to work with vectors and matrices. Actually, we can work with matrices whose entries come from any set that satisfies these properties, such as the set of all rational numbers or the set of all complex numbers.

(i) *Closure.* For all real numbers a, b, the sum $a + b$ and the product $a \cdot b$ are real numbers.

(ii) *Associative laws.* For all real numbers a, b, c,

$$a + (b + c) = (a + b) + c \quad \text{and} \quad a \cdot (b \cdot c) = (a \cdot b) \cdot c \,.$$

(iii) *Commutative laws.* For all real numbers a, b,

$$a + b = b + a \quad \text{and} \quad a \cdot b = b \cdot a \,.$$

(iv) *Distributive laws.* For all real numbers a, b, c,

$$a \cdot (b + c) = a \cdot b + a \cdot c \quad \text{and} \quad (a + b) \cdot c = a \cdot c + b \cdot c \,.$$

(v) *Identity elements.* There are real numbers 0 and 1 such that for all real numbers a,

$$a + 0 = a \quad \text{and} \quad 0 + a = a \,,$$

$$a \cdot 1 = a \quad \text{and} \quad 1 \cdot a = a \,.$$

(vi) *Inverse elements.* For each real number a, the equations

$$a + x = 0 \quad \text{and} \quad x + a = 0$$

have a solution x in R, called the *additive inverse* of a, and denoted by $-a$.

For each nonzero real number a, the equations

$$a \cdot x = 1 \quad \text{and} \quad x \cdot a = 1$$

have a solution x in R, called the *multiplicative inverse* of a, and denoted by a^{-1}.

Here are some additional properties of real numbers a, b, c, which can be proved from the properties listed above.

(a) If $a + c = b + c$, then $a = b$.
(b) If $a \cdot c = b \cdot c$ and $c \neq 0$, then $a = b$.
(c) $a \cdot 0 = 0$
(d) $-(-a) = a$
(e) $(-a) \cdot (-b) = a \cdot b$